Problem 1

Show that if $u(x,t)$ and $v(x,t)$ are solutions to the Dirichlet problems for the Heat equation

$$u_t(x,t) - ku_{xx}(x,t) = f(x,t), \quad u(x,0) = \phi_1(x), u(0,t) = u(1,t) = g_1(t)$$
$$v_t(x,t) - kv_{xx}(x,t) = f(x,t), \quad v(x,0) = \phi_2(x), v(0,t) = v(1,t) = g_2(t),$$

and if $\phi_2(x) \le \phi_1(x)$ for $0 \le x \le 1,\ g_2(t) \le g_1(t),\ t > 0$, then for all $0 < x < 1,\ t > 0$, we have $u(x,t) \ge v(x,t)$.

Problem 2

Show the uniqueness for the equation $u_t(x,t) - ku_{xx}(x,t) = f(x,t)$ with the mixed boundary conditions $u(x,0) = \phi(x),\ u(0,t) = g(t),\ u_x(l,t) = h(t)$.
Hint: Use the Energy method.

Problem 3

Find the solution of the Heat equation $u_t = ku_{xx}$ with the initial condition $u(x,0) = \sinh x$.

Problem 4

Find the solution of the Heat equation $u_t = ku_{xx}$ with the initial condition

$$u(x,0) = \begin{cases} 2, & -3 < x < 1 \\ 1, & 1 < x < 3 \\ 0, & |x| > 3. \end{cases}$$

Problem 5

Find a solution of the heat equation with convection

$$\begin{cases} u_t - ku_{xx} + Vu_x = 0 \\ u\left(x,0\right) = \phi\left(x\right) \end{cases}$$

Hint: use the substitution

$$\begin{cases} \tau = t \\ \zeta = x - Vt \end{cases}$$

and the formula for the solution of the homogeneous diffusion equation.

Due date: October 11, 2012