Problem 1

Solve the Heat equation $u_t = ku_{xx}$ for $0 < x < \pi$, $t > 0$ with the initial condition

$$u(x, 0) = 1 + 2\sin x$$

and the boundary conditions $u(0, t) = u(\pi, t) = 1$.
Hint: Notice that the boundary condition is not homogeneous.

Problem 2

Find all the separated solutions of the equation $tu_t - ku_{xx} + u = 0$ subject to the boundary conditions $u(0, t) = u_x(\pi, t) = 0$. Use them to find all the solutions of the initial value problem $u(x, 0) = 0$.

Problem 3

Find the solution of the Wave equation $u_{tt} = 4u_{xx}$ with

$$u(0, t) = u(\pi/2, t) = 0,\ t > 0,$$

$$u(x, 0) = \sin(2x) - 2\sin(6x),\ u_t(x, 0) = -3\sin 4x\ 0 \le x \le \pi/2.$$

Problem 4

Find the solution of the Heat equation $u_t = 4u_{xx}$ with

$$u_x(0, t) = u_x(\pi/2, t) = 0,\ t > 0, u(x, 0) = 3\sin^2 x,\ 0 \le x \le \pi/2.$$

Hint: Use Trigonometric identities.

Problem 5

Find a polynomial $p(x)$ which together with 1 and x form an orthogonal basis of the space of all polynomials of degree at most 2 on the interval $[-1, 1]$ with the usual scalar product

$$< f, g >= \int_{-1}^{1} f(x)g(x)\, dx.$$

Is the basis orthonormal?

Due date: November 1, 2012