Problem 1

Show that the following identity holds for any two functions f, g in $L^2[a, b]$:

$$\|f + g\|_2^2 + \|f - g\|_2^2 = 2(\|f\|_2^2 + \|g\|_2^2).$$

Problem 2

Find the Fourier Sine Series of x^2 on $[0, \pi]$.

Problem 3

Let $e_n(x) = \sin \pi(n + 1/2)x$. Check that $(e_n(x))_{n \geq 0}$ form an orthogonal system on $[0, 1]$.

Problem 4

Find the Fourier series of $f(x) = x$ in the basis from the previous problem. (You do not have to prove that $(e_n(x))_{n \geq 0}$ form a basis!)

Problem 5

Show that the orthogonal family $(e_n(x) = \cos nx)_{n \geq 1}$ *do not* form an orthogonal basis in $L^2[0, \pi]$.

Due date: November 8, 2012