

Problem 1

Any function ϕ defined on a symmetric interval $[-l, l]$ can be (uniquely) written as $\phi = \phi_e + \phi_o$ where ϕ_e is an even function and ϕ_o is an odd function. Find ϕ_e and ϕ_o in terms of ϕ . (**Hint:** you should write the above equality at x and at $-x$). If the full Fourier series expansion of ϕ on $[-l, l]$ is

$$\phi(x) = A_0 + \sum_{n=1}^{\infty} \left(A_n \cos \frac{n\pi x}{l} + B_n \sin \frac{n\pi x}{l} \right)$$

then what are the full Fourier series expansions of ϕ_e and ϕ_o on $[-l, l]$?

Problem 2

Let d_n be a sequence of real numbers. Let f_n , $n \geq 1$ be functions defined on $[0, 1]$ by

$$f_n(x) = \begin{cases} d_n, & \frac{1}{3} - \frac{1}{n+2} \leq x < \frac{1}{3}, \\ 0 & \text{otherwise.} \end{cases}$$

Find the pointwise limit f of the sequence $\{f_n\}$.

Problem 3

Give an example of a sequence (d_n) such that the sequence (f_n) from the previous problem converges to the function f from the previous problem in the L^2 sense but at the same time f_n^2 doesn't converge to f^2 in the L^2 sense.

Problem 4

Let $f(x)$ be a continuous function on $[-l, l]$ and let $F(x) = \int_{-l}^x f(s) ds$. If the full Fourier series of f on $[-l, l]$ is

$$0 + \sum_{n=1}^{\infty} \left(A_n \cos \frac{n\pi x}{l} + B_n \sin \frac{n\pi x}{l} \right)$$

what is the full Fourier series of F on $[-l, l]$? The purpose of this question is to check that it is possible to do term by term integration, so you should answer the question by computing the coefficients for the full series of F in terms of the coefficients of f . (**Hint:** write the formulas for the coefficients and integrate by parts).

Problem 5

Find the solution of the Heat Equation $u_t = k u_{xx}$ with the boundary conditions $u(0, t) = u(l, t) = 0$ and the initial condition $u(x, 0) = x^2$.

Due date: November 15, 2012