Problem 1

Prove *Greens first identity*: For every pair of functions $X_1(x), X_2(x)$ on (a,b),

$$\int_a^b f''(x)\, g(x)\, dx = f'(x)\, g(x)\Big|_a^b - \int_a^b f'(x)\, g'(x)\, dx.$$

Problem 2

Use the previous problem to show that the following eigenvalue problem has no negative eigenvalues

$$\begin{cases} X''(x) + \lambda X(x) = 0 \\ X(x)\, X'(x)\,|_a^b \leq 0. \end{cases}$$

Problem 3

Show that the eigenvalue problem

$$\begin{cases} X''(x) + \lambda X(x) = 0 \\ X(a) = 2X(b),\ X'(b) = 2X'(a). \end{cases}$$

has no negative eigenvalues.

Problem 4

Find all harmonic functions in the annulus $\left\{(x,y):\ 1 < (x-1)^2 + (y+1)^2 < 2\right\}$ which depend only on $(x-1)^2 + (y+1)^2$.

Problem 5

Let $f = (u,v) : D_1 \subset \mathbb{R}^2 \to D_2 \subset \mathbb{R}^2$ be a mapping such that in D_1 it satisfies $u_x = v_y$ and $u_y = -v_x$. Show that if ϕ is a harmonic function in D_2 then $\phi \circ f$ is a harmonic function in D_1.

Due date: November 22, 2012