Assignment 5, due February 11

Problem 1

Let $x = p_1 p_2 \dots p_k$, where p_1, p_2, $\dots$,p_k are different prime numbers. Let also $(p_j - 1)|(x - 1)$ for $j = 1, 2, \dots, k$. Show that x is a Carmichael number.

Problem 2

Suppose that $k \in \mathbb{N}$ and the numbers $6k + 1$, $12k + 1$, and $18k + 1$ are all prime. Show that $(6k + 1)(12k + 1)(18k + 1)$ is a Carmichael number.

Problem 3

Find a recurrent formula for the solution of the congruency

$$6x \equiv 1 (\bmod 11^e),$$

where $e \in \mathbb{N}$.

Problem 4

Find a recurrent formula for the solution of the congruency

$$x^3 \equiv 1 (\bmod 5^e),$$

where $e \in \mathbb{N}$.

Problem 5

Show that $p \geq 3$ is a prime if and only if

$$2(p - 3)! \equiv -1 (\bmod p).$$