MAT334, Midterm Practice Problems

Problem 1.

Compute $(\sqrt{2}/2 + i\sqrt{2}/2)^{239}$.

Problem 2.

Compute $\exp(\ln 2 + \pi i)$.

Problem 3.

Find all the solutions of the equation $z^5 = 32$.

Problem 4.

Find $\lim_{z \to 0} \frac{e^z - e^{-z}}{z}$

Problem 5.

Show that the function $u(x, y) = \sin x(e^y + e^{-y})$ is harmonic, and find its harmonic conjugate.

Problem 6.

Show that the function $f(x + iy) = x^4 + y^4 + 1 - 2x^2 + 2y^2 - 6x^2y^2 + i(4x^3y - 4xy^3 - 4xy)$ is entire, and find its derivative.

Problem 7.

For a function $f(z) = \frac{z+i}{z-i}$ find the level curve $|f(z)| = 1$.

Problem 8.

Write the polynomial $z^4 - z + 1$ at the Taylor form centered at i.

Problem 9.

Find all the poles of the rational function $f(z) = \frac{z+i}{z^4-z}$.

Problem 10.

Find $\lim_{n \to \infty} \frac{\sinh(in)}{n}$.

Answers

Problem 1.

$\sqrt{2}/2 - i\sqrt{2}/2$.

Problem 2.

-2.

Problem 3.

2, $2e^{2\pi i/5}$, $2e^{4\pi i/5}$, $2e^{6\pi i/5}$, $2e^{8\pi i/5}$.

Problem 4.

2 - use L'Hospital rule.

Problem 5.

$\cos x(e^y - e^{-y})$.

Problem 6.

$f' = 4x^3 - 4x - 12xy^2 + i(12x^2y - 4y^3 - 4y)$.

Problem 7.

$Im z = 0$.

Problem 8.

$2 - i - (4i + 1)(z - i) - 6(z - i)^2 + 4i(z - i)^3 + (z - i)^4$.

Problem 9.

0, 1, $e^{2\pi i/3}$, $e^{4\pi i/3}$.

Problem 10.

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