Complex Analysis

Assignment 7, due March 25

Problem 1 of 5 (A version of Weierstrass convergence theorem.). Let (f_n) be a sequence of functions analytic in a region Ω. Assume that $\sum_{n=1}^{\infty} |f_n(z)|$ converges locally uniformly in Ω. Show that for any k, the series of k-th derivatives, $\sum_{n=1}^{\infty} |f_n^{(k)}(z)|$ also converges locally uniformly in Ω.

Problem 2 of 5. Let f be a function analytic in the closed unit disk. Show that there exists $M > 0$ such that for any $A \in \mathbb{C}$ with $|A| > M$, the equation $f(z) = Az^n$ has exactly n solutions in the unit disk counted with multiplicity.

Problem 3 of 5. Let $p(z) = z^d + a_{d-1}z^{d-1} + \dots a_0$ be a polynomial, $d \geq 1$. Show that there exists c such that $|c| = 1$ and $|p(c)| \geq 1$.

Problem 4 of 5. Let f be a non-constant function analytic in a region Ω, and the sequence of the functions (f_n), analytic in Ω, converge to f locally uniformly. Let $f(z_0) = 0$. Show that there exists a sequence $z_n \to z_0$ such that for any n, $f_n(z_n) = 0$.

Problem 5 of 5. Problem 3, parts (b), (d), (f), (g), (h); page 161 of *Ahlfors*.