

HOMEWORK #1
DUE DATE: JANUARY 14

Problem 1. Describe all the functions which are also equivalence relations.

Problem 2. Let $f : A \rightarrow X$ be a function, and let $Y, Z \subset X$. Prove that

$$f^{-1}(Y \cup Z) = f^{-1}(Y) \cup f^{-1}(Z).$$

Problem 3. Suppose that $f : A \rightarrow B$ and $g : B \rightarrow A$ satisfy $g \circ f = id_A$. Show that f is injective and g is surjective.

Problem 4. Prove that $\log_2 5$ is an irrational number.

Problem 5. Define a relation R on the space X of all real-valued functions on $[0, 1]$ by

$$(f, g) \in R \text{ if and only if } f(0) = g(0) \text{ and } f(1) = g(1).$$

Show that R is an equivalence relation and that the equivalence classes form a two-dimensional vector space.

Hint: Check that addition and multiplication by real numbers are well defined on the equivalence classes.