NAME (PRINT): ______________________________
Last/Surname First /Given Name

STUDENT NO: ______________________________ **SIGNATURE:** ______________________________

UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2008 FINAL EXAMINATION
MAT406H5F
Mathematical Introduction to Game Theory
Ilia Binder
Duration - 3 hours
Aids: one page of single-sided Letter sheet.

You may be charged with an academic offense for possessing the following items during the writing of an exam unless otherwise specified: any unauthorized aids, including but not limited to calculators, cell phones, pagers, wristwatch calculators, personal digital assistants (PDAs), iPods, MP3 players, or any other device. If any of these items are in your possession in the area of your desk, please turn them off and put them with your belongings at the front of the room before the examination begins. A penalty may be imposed if any of these items are kept with you during the writing of your exam.

Please note, students are **NOT** *allowed to petition to* **RE-WRITE** *a final examination.*

Qn. #	Value	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Consider the following game. Two players start with a pile of chips. Players take turn removing 2 or 5 chips from the pile. The player who can not make a move loses the game. Describe, with an explanation, all P and N positions.

10 points

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10 points

2. Now the game from the previous problem is played with three piles of chips, and players take turn removing 2 or 5 chips from one of the piles. Which of the players has the winning strategy if the starting position is $(239, 23, 4)$? Explain your answer.

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3. Consider the following game. Two players start with a pile of chips. Players take turn removing chips from the pile. The first player is allowed to remove any non-zero even number of chips, while the second player can remove any non-zero number of chips divisible by 3. The player who can not make a move wins the game (thus, the game is played under misere rule). Describe, with an explanation, all the starting positions where the first player has a winning strategy.

10 points

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4. Solve (i.e. find the value of the game and optimal strategies for both players) the two-person zero sum game given in strategic form by the following matrix.

$$\begin{pmatrix} -1 & -1 \\ -5 & 5 \\ 4 & -1 \\ 0 & 0 \\ 3 & -2 \end{pmatrix}$$

10 points

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5. Solve (i.e. find the value of the game and optimal strategies for both players) the two-person zero sum game given in strategic form by the following matrix.

$$\begin{pmatrix} 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ -1 & 0 & 0 & -1 & 2 \\ -1 & 1 & -1 & 1 & 0 \end{pmatrix}$$

10 points

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6. Solve (i.e. find the value of the game and optimal strategies for both players) the two-person zero sum game given in strategic form by the following matrix.

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

10 points

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7. Consider the following game. Each of the two players tosses a fair coin. The outcomes are not shown to the opposing player. The first player can put \$1 on the table or fold (and thus end the game without any payoffs). The second player then can either call (and put \$1 on the table) or fold (and thus again end the game with no payoff). If neither of the players folds the first player takes the money on the table if the both tossed coins gave the same value. Otherwise, this amount is taken by the second player.

10 points Draw the Kuhn tree of the game, convert it to strategic form, and solve.

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8. Find all Nash Equilibria for the game given in the matrix form by the following bi-matrix.

$$\begin{pmatrix} (1,4) & (4,1) \\ (5,2) & (1,-1) \end{pmatrix}$$

10 points

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9. Consider the following model of duopoly.

   The market has capacity A for a certain good. The production cost of each unit equal to C_1 for the company I and $C_2 < C_1$ for the company II. The price of the product is equal to $A - Q$, where Q is the total number of the units produced. Company I makes a decision about the number of units it will produce and informs Company II about its decision. The company II then makes the decision about the number of units it will produce.

   10 points Analyze the model by finding all Nash equilibria, and comparing the consumer prices and profits with the case of the monopoly of the company I.

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10. Find the point of the optimal agreement for the two-person cooperative TU game given by the following bi-matrix.

$$\begin{pmatrix} (2,0) & (3,3) & (2,1) & (10,9) & (8,8) \\ (4,4) & (5,4) & (3,3) & (2,1) & (3,2) \\ (2,3) & (0,0) & (1,1) & (4,5) & (6,4) \\ (-1,0) & (8,7) & (5,6) & (3,2) & (2,2) \end{pmatrix}$$

10 points

Hint: You have already solved half of this problem during this test.

Total Marks = 100 points