NAME (PRINT): _____
Last/Surname First/Given Name

STUDENT #: _____ **SIGNATURE:** _____

UNIVERSITY OF TORONTO MISSISSAUGA
April 2014 FINAL EXAMINATION
MAT406H5S
Mathematical Introduction to Game Theory
Ilia Binder
Duration – 2 hours
Aids: 1 page(s) of single-sided Letter (8-1/2 x 11) sheet

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam. Clear, sealable, plastic bags have been provided for all electronic devices with storage, including but not limited to: cell phones, tablets, laptops, calculators, and MP3 players. Please turn off all devices, seal them in the bag provided, and place the bag under your desk for the duration of the examination. You will not be able to touch the bag or its contents until the exam is over.

If, during an exam, any of these items are found on your person or in the area of your desk other than in the clear, sealable, plastic bag; you may be charged with an academic offence. A typical penalty for an academic offence may cause you to fail the course.

Please note, you **CANNOT** *petition to* **re-write** *an examination once the exam has begun.*

Qn. #	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1 (20points). In a splitting game, the players are given a few non-empty piles of stones. A legal move consists of splitting a pile into two non-empty piles. The winner makes the last move. Thus the terminal positions consist only of a few piles of size one.

Compute SG-function of the game and use it to determine which starting one-pile positions are winning for the first player to move. Justify your answer.

continued on page 3

continued on page 4

Problem 2 (20points). Solve (i.e. find the value of the game and optimal strategies for both players) the two-person zero sum game given in strategic form by the following matrix.

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

Hint: Use domination and Principle of indifference.

continued on page 5

continued on page 6

Problem 3 (20points). Players I and II are given a card at random. Each card is a *Winning* card with probability 3/4, and a *Losing* card with probability 1/4. After looking at their cards, without seeing the card of the other player, each player try to guess the card of another player. If players have the same cards (both Winning or both Losing), the player guessing correctly is paid \$1, the player guessing incorrectly looses \$1. Otherwise, there is no payoff.

(1) Draw the Kuhn tree.
(2) Find the equivalent strategic form.
(3) Find the safety levels.
(4) Find all Nash Equilibria.

continued on page 7

continued on page 8

Problem 4 (20points). Consider a two-person cooperative game given by the following matrix

$$\begin{pmatrix} (3,5) & (6,2) & (4,3) & (5,0) \\ (2,6) & (1,0) & (0,3) & (3,2) \\ (0,7) & (2,7) & (7,3) & (4,8) \\ (-1,11) & (-1,2) & (1,3) & (2,4) \end{pmatrix}.$$

(1) Find all Pareto-optimal strategies
(2) Solve the game as a TU game.
(3) Find a λ-transfer solution assuming it is an NTU game.

continued on page 9

continued on page 10

Problem 5 (20points). A toy costs \$100 and consists of three parts: I, II, III. There is one manufacturer of part I, two manufacturers of part II, and three manufacturers of part III.

(1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
(2) Describe all the imputations.
(3) Compute the core of the game.
(4) Find the Shapley Value.

continued on page 11

Total Marks = 100 points