MAT406H5F. Assignment 6, due November 16

Problem 1 of 5

Let us consider a general sum $m \times n$ game.

a) Prove that for a fixed $\mathbf{p} \in \Delta^m$ the set of points

$$\{\mathbf{q} \in \Delta^n : \ (\mathbf{p}, \mathbf{q}) \text{ is a Nash equilibrium.}\}$$

is convex.

b) Show that the set

$$\{(\mathbf{p}, \mathbf{q}) \in \Delta^m \times \Delta^n : \ (\mathbf{p}, \mathbf{q}) \text{ is a Nash equilibrium.}\}$$

does not have to be convex.

Problem 2 of 5

Consider the 3×3 two-person non-zero-sum game with payoff matrix:

$$\begin{pmatrix} (1,0) & (-1,0) & (0,1) \\ (0,0) & (1,1) & (-1,-3) \\ (0,2) & (0,-2) & (5,4) \end{pmatrix}$$

Find a Nash equilibrium.

Problem 3 of 5

Consider the 3×3 two-person non-zero-sum game with payoff matrix:

$$\begin{pmatrix} (3,2) & (3,0) & (2,2) \\ (1,0) & (2,3) & (0,3) \\ (0,2) & (0,0) & (3,2) \end{pmatrix}$$

Find all the Nash equilibria (pure and mixed) of the game.

Problem 4 of 5

Karlin-Peres, Problem 4.16

Problem 5 of 5

Consider the following model of duopoly.
The market has capacity A for a certain good. The production cost of each unit equal to C_1 for the company I and $C_2 < C_1$ for the company II. The price of the product is equal to $A - Q$, where Q is the total number of the units produced. Company I makes a decision about the number of units it will produce and informs Company II about its decision. The company II then makes the decision about the number of units it will produce.
Analyze the model by finding all Nash equilibria, and comparing the consumer prices and profits with the case of the monopoly of the company I.