MAT406H5F. Assignment 8, due November 30

Problem 1 of 5

Let (N, v_1) and (N, v_2) be two games in coalitional form with non-empty cores. Prove that $v_1 + v_2$ is a characteristic function and the game $(N, v_1 + v_2)$ has a non-empty core.

Problem 2 of 5

Find the characteristic function of the 3-person game in strategic form when the payoff vectors are:

If I chooses 1:

$$\begin{pmatrix} (2,7,-2) & (3,0,1) \\ (-1,6,3) & (3,-2,1) \end{pmatrix}$$

If I chooses 2:

$$\begin{pmatrix} (-1,2,4) & (1,3,3) \\ (7,5,-4) & (3,-2,1) \end{pmatrix}$$

Problem 3 of 5

(**Oil Market game.**) Country 1 has oil which it can use to run its transport system at a profit of a per barrel. Country 2 wants to buy the oil to use in its manufacturing industry, where it gives a profit of b per barrel, while Country 3 wants it for food manufacturing where the profit is c per barrel. Let $a < b \leq c$.

1. Describe the problem as a game in coalitional form, i.e. define the characteristic function.
2. Describe all the imputations.
3. Compute the core of the game.
4. Find the Shapley Value.

Problem 4 of 5

Consider a weighted majority game with four players with the weights $5, 10, 10, 22$.

1. Write the game in coalition form.
2. Find the core of the game.
3. Compute the Shapley-Shubik power index.

Problem 5 of 5

Prove that the game in coalitional form (N, v) is inessential if and only if $-v$ is a characteristic function.