NAME (PRINT):

Last/Surname

First/Given Name

STUDENT #:

SIGNATURE: \_\_\_\_

## UNIVERSITY OF TORONTO MISSISSAUGA DECEMBER 2015 FINAL EXAMINATION MAT406H5F Mathematical Introduction to Game Theory Ilia Binder Duration - 2 hours Aids: 1 page(s) of single-sided Letter (8-1/2 x 11) sheet

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam. Clear, sealable, plastic bags have been provided for all electronic devices with storage, including but not limited to: cell phones, SMART devices, tablets, laptops, calculators, and MP3 players. Please turn off all devices, seal them in the bag provided, and place the bag under your desk for the duration of the examination. You will not be able to touch the bag or its contents until the exam is over.

If, during an exam, any of these items are found on your person or in the area of your desk other than in the clear, sealable, plastic bag; you may be charged with an academic offence. A typical penalty for an academic offence may cause you to fail the course.

Please note, you **CANNOT** petition to **re-write** an examination once the exam has begun.

Qn. #	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

**Problem 1 (20points)**. Consider the following game. Two players start with a pile of chips. Players take turns removing 2, 3, or 5 chips from the pile. The player who cannot make a move loses the game. Compute the SG-function and describe all P- and N- positions. Justify your answer.

Problem 1 (20points)

**Problem 2 (20points)**. Solve (i.e. find the value of the game and optimal strategies for both players) the two-person zero sum game given by the following matrix.

$$\begin{pmatrix} 0 & 2 & 1 & 2 & -1 \\ -1 & 1 & 0 & 0 & 2 \\ 1 & 2 & -2 & 2 & 0 \\ -2 & -2 & -1 & 3 & 1 \end{pmatrix}$$

Problem 2 (20points)

**Problem 3 (20points)**. Find the safety levels, the maxmin strategies, and all of the Nash Equilibria for the game given in the matrix form by the following bi-matrix.

$$\begin{pmatrix} (1,6) & (2,9) & (3,4) & (2,8) \\ (5,4) & (0,5) & (4,3) & (6,3) \\ (2,5) & (3,6) & (5,4) & (3,7) \end{pmatrix}$$

Problem 3 (20points)

**Problem 4 (20points)**. Find the point of the optimal agreement for the two-person cooperative TU game given by the following bi-matrix.

(2,2)	(3,1)	(2, 1)	(10, 8)	(8,9)
(4,5)	(5, 4)	(3,3)	(1, 1)	(3,1)
(4, 3)	(2, 0)	(-3, -1)	(4, 2)	(4, 4)
$\setminus (-1,1)$	(5,7)	(5,6)	(3,0)	(2,1)

Hint: You have already solved a half of this problem during this test.

Problem 4 (20points)

Problem 5 (20points). Consider a weighted majority game with four players with the weights 1, 3, 3, 5.

- (1) Describe the set of imputations of the game.
- (2) Describe the core of the game.
- (3) Compute the Shapley-Shubik power index.

Problem 5 (20points)