NAME (PRINT): ____

Last/Surname First/Given Name

STUDENT #: ____________________ **SIGNATURE:** ____________

UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2016 FINAL EXAMINATION
MAT406H5F
Mathematical Introduction to Game Theory
Ilia Binder
Duration – 2 hours
Aids: 1 page(s) of single-sided Letter (8-1/2 x 11) sheet

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Please note, once this exam has begun, you **CANNOT** *re-write it.*

Qn. #	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1 (20points). In a splitting game, the players are given a few non-empty piles of stones. A legal move consists of splitting a pile into two non-empty piles. The winner makes the last move. Thus the terminal positions consist only of a few piles of size one.

Using the SG theorem, compute the SG-function of the game and use it to determine which starting one-pile positions are winning for the first player to move. Justify your answer.

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Problem 2 (20points). Solve (i.e. find the value of the game and all the optimal strategies for both players) the two-person zero sum game given in strategic form by the following matrix.

$$\begin{pmatrix} -1 & 0 & 0 & -2 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & -2 \end{pmatrix}$$

Hint: Use domination and the Principle of indifference.

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Problem 3 (20points). Players I and II are given a card at random. Each card is a *Winning* card with probability 1/3, and a *Losing* card with probability 2/3. After looking at their cards, without seeing the card of the other player, each player tries to guess the card of the other player. If players have the same cards (both Winning or both Losing), each player who made a correct guess is paid \$2 and each player who made an incorrect guess loses \$1. Otherwise, each player who made a correct guess is paid \$1 and each player who made an incorrect guess loses \$1.

(1) Draw the Kuhn tree.
(2) Find the equivalent strategic form.
(3) Find the safety levels.
(4) Find all Nash Equilibria.

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Problem 4 (20points). Consider a two-person cooperative game given by the following matrix

$$\begin{pmatrix} (4,0) & (1,-1) & (12,-1) & (10,-9) & (0,0) \\ (7,5) & (3,1) & (3,2) & (2,1) & (-1,2) \\ (2,3) & (0,0) & (1,1) & (4,5) & (-1,4) \\ (-1,0) & (8,7) & (5,6) & (3,2) & (-1,5) \end{pmatrix}.$$

(1) Find all Pareto-optimal strategies
(2) Solve the game as a TU game.
(3) Find a λ-transfer solution assuming it is an NTU game.
Hint: It is easier to use the Nash solution here with a suitably chosen threat point.

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Problem 5 (20points). A toy costs \$40 and consists of three parts: I, II, III. There is one manufacturer of part I, two manufacturers of part II, and three manufacturers of part III.

(1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
(2) Describe all the imputations.
(3) Compute the core of the game.
(4) Find the Shapley Value.

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Total Marks = 100 points