Mathematical Introduction to Game Theory Assignment 2, due September 26

Problem 1 of 5. Find the Sprague-Grundy function for the subtraction game with subtraction set $\{1, 4, 5\}$ and normal winning conditions. Justify your answer.

Problem 2 of 5. Consider the take-away game with normal winning conditions and the rule that you may remove any number of chips not divisible by five. Find the formula for Sprague-Grundy function.

Problem 3 of 5. The game is played by the following rules. There are four piles of chips. Players can take any number of chips from any of the first two piles or any number of chips not divisible by five from the third or fourth pile. The last player to move wins. Find the Sprague-Grundy function of the initial position (23, 45, 17, 48). If the winning moves in this position exist, find all of them. Justify your answer.

Hint: Don't forget that Sprague-Grundy function might actually *increase* after the move.

Problem 4 of 5. A crippled queen game is played on the board of the size $n \times 2$, $n \in \mathbb{N}$. Find a formula for Sprague-Grundy function. Justify your answer.

Problem 5 of 5. Consider a partial subtraction game with the subtraction set for the first player $S_1 = \{1, 2\}$, and the subtraction set for the second player $S_2 = \{1, 3, 4\}$. Find all the winning positions for the first player. Justify your answer.