# Mathematical Introduction to Game Theory
## Assignment 3, due October 3

**Problem 1 of 5.** The **Game of** $Y$ is played on a triangular board tiled with hexagons. As in Hex, the two players take turns coloring in hexagons, each using his assigned color. A player wins when he establishes a $Y$, a monochromatic connected region in his color that meets all three sides of the triangle. Please look at Karlin-Peres book, figure 1.12, for a picture.

Prove that the first player has a winning strategy.

**Problem 2 of 5.** Consider a zero-sum game with the matrix

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 \\ 1 & -2 & 0 & -1 \\ -2 & 0 & 0 & 1 \end{pmatrix}.$$

Find Chris' optimal response to the $\left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, 0\right)$ strategy of Ruth.

**Problem 3 of 5.** Consider a zero-sum game with the matrix

$$\begin{pmatrix} 0 & 1 & 1 & a \\ -1 & 0 & 2 & 0 \\ 1 & -2 & 0 & -2 \\ -2 & 0 & 0 & -1 \end{pmatrix}.$$

where $a$ is a parameter. For which values of $a$ does the game have saddle point?

**Problem 4 of 5.** Solve the following zero-sum game, i.e. find the value of the game and all optimal strategies for both players

$$\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}.$$

**Problem 5 of 5.** Find all the values of $x$ for which the game with the matrix

$$\begin{pmatrix} -x & 2 \\ -1 & x \end{pmatrix}$$

has a strictly positive value.