Mathematical Introduction to Game Theory

Assignment 5, due November 7

Problem 1 of 5. For the following game, find the safety levels of both players, all Pareto optimal strategies, and all pure strategic equilibria

$$\begin{pmatrix} (0,2) & (1,-1) & (2,1) \\ (-1,3) & (2,-1) & (0,2) \\ (2,2) & (-1,2) & (4,1) \\ (1,1) & (2,2) & (2,0) \end{pmatrix}.$$

Problem 2 of 5. At the beginning of a game, Ruth and Chris are given \$100 and \$200 dollars respectively. Each must decide to pass or gamble, not knowing the choice of the other. A player who passes keeps the money he/she started with. If Ruth gambles, she wins additional \$200 with probability 1/2 or loses all her money with probability 1/2. If Chris gambles, he wins or loses \$200 with probability 1/2 each. These outcomes are independent. In addition, the contestant with the higher amount of money at the end wins a bonus of \$300.

(1) Draw the Kuhn tree.

(2) Convert to strategic form.

(3) Find the safety levels.

(4) Find all the strategic equilibria.

Problem 3 of 5. Prove that in a two-person general sum game, the expected payoff of any player at any Strategic Equilibrium (mixed or pure) can not be smaller than the safety level of this player.
Hint: A player can always switch to his/her optimal strategy if this would not be the case.

Problem 4 of 5. Find all the Nash equilibria in the game with the matrix

$$\begin{pmatrix} (0,4) & (3,0) \\ (2,2) & (1,3) \end{pmatrix}.$$

Problem 5 of 5. Let (A,B) be a *constant-sum game*, i.e. there exists a constant L such that for every i, j, $a_{ij} + b_{ij} = L$. Prove that the payoff of R is the same at any Nash equilibrium. Prove that the same is true for C.
Hint: If $L = 0$, it is a zero-sum game, and we can use Minimax Theorem. For other L, just subtract it from all elements of one of the matrices.