

Mathematical Introduction to Game Theory

Assignment 8, due November 28

Problem 1 of 5. Let (N, v_1) and (N, v_2) be two games in coalitional form with non-empty cores. Prove that $v_1 + v_2$ is a characteristic function and the game $(N, v_1 + v_2)$ has a non-empty core.

Problem 2 of 5. Find the characteristic function of the 3-person game in strategic form when the payoff vectors are:

If I chooses 1:

$$\begin{pmatrix} (2, 7, -2) & (3, 0, 1) \\ (-1, 6, 3) & (3, -2, 1) \end{pmatrix}$$

If I chooses 2:

$$\begin{pmatrix} (-1, 2, 4) & (1, 3, 3) \\ (7, 5, -4) & (3, -2, 1) \end{pmatrix}$$

Problem 3 of 5. (Oil Market game.) Country 1 has oil which it can use to run its transport system at a profit of a per barrel. Country 2 wants to buy the oil to use in its manufacturing industry, where it gives a profit of b per barrel, while Country 3 wants it for food manufacturing where the profit is c per barrel. Let $a < b \leq c$.

- (1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
- (2) Describe all the imputations.
- (3) Compute the core of the game.
- (4) Find the Shapley Value.

Problem 4 of 5. A toy costs \$100 and consists of three parts: I , II , III . There is one manufacturer of part I , two manufacturers of part II , and three manufacturers of part III .

- (1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
- (2) Describe all the imputations.
- (3) Compute the core of the game.
- (4) Find the Shapley Value.

Problem 5 of 5. Prove that the game in coalitional form (N, v) is inessential if and only if $-v$ is a characteristic function.