NAME (PRINT): ____
Last/Surname   First/Given Name

STUDENT #: ____________________ **SIGNATURE:** ____________

UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2017 FINAL EXAMINATION
MAT406H5F
Mathematical Introduction to Game Theory
Ilia Binder
Duration - 2 hours
Aids: 1 page(s) of single-sided Letter (8-1/2 x 11) sheet

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Please note, once this exam has begun, you ***CANNOT*** *re-write it.*

Qn. #	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1 (20points). Consider the following game. Two players start with a few piles of chips. A legal move consists of removing 1, 3, or 4 chips from one of the piles. The player who cannot make a move loses the game. Compute the SG-function for all one-pile starting positions. Is the position $(5, 11, 34)$ $P-$ or $N-$position? If it is an $N-$position, describe all the winning moves. Justify your answer.

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Problem 2 (20points). Player I draws a card at random from a full deck of 52 cards. After looking at the card, he bets either 1 or 3 that the card he drew is a face card (king, queen or jack, probability 3/13). Then Player II either concedes or doubles. If she concedes, she pays I the amount bet (no matter what the card was). If she doubles, the card is shown to her, and Player I wins twice his bet from the player II if the card is a face card, and loses twice his bet to the player II otherwise.

(1) Draw the Kuhn tree.
(2) Find the equivalent strategic form.
(3) Solve the game.

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Problem 3 (20points). Find the safety levels, a maxmin strategy for each player, and all Nash Equilibria for the game given in the matrix form by the following bi-matrix.

$$\begin{pmatrix} (1,4) & (2,7) & (3,2) & (2,6) \\ (5,2) & (0,3) & (4,1) & (6,1) \\ (2,3) & (3,4) & (5,4) & (3,5) \end{pmatrix}$$

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Problem 4 (20points). Consider a two-person cooperative game given by the following bi-matrix

$$\begin{pmatrix} (2,0) & (3,-3) & (2,-1) & (10,-7) & (0,0) \\ (7,5) & (3,1) & (3,2) & (2,1) & (-1,2) \\ (2,3) & (0,0) & (1,1) & (4,5) & (-1,4) \\ (-1,0) & (8,7) & (5,6) & (3,2) & (-1,5) \end{pmatrix}.$$

(1) Find all pure Pareto-optimal strategies.
(2) Solve the game as a TU game.
(3) Find the λ-transfer solution assuming it is an NTU game. You just need to provide the optimal agreement, not the value of λ.

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Problem 5 (20points). Consider a weighted majority game with five players with the weights 1, 5, 10, 10, 22.

(1) Describe the set of imputations of the game.
(2) Describe the core of the game.
(3) Compute the Shapley-Shubik power index.

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Total Marks = 100 points