Mathematical Introduction to Game Theory

Assignment 2, due September 25

Problem 1 of 5. Consider the take-away game with the rule that you may remove any number of chips not divisible by four. Find the formula for Sprague-Grundy function.

Problem 2 of 5. A crippled queen game is played on the board of the size $2 \times n$, $n \in \mathbb{N}$. Find a formula for Sprague-Grundy function. Justify your answer.

Problem 3 of 5. The game is played by the following rules. There are four piles of chips. Players can take any number of chips from any of the first two piles or any number of chips not divisible by three from the third or fourth pile. Find the Sprague-Grundy function of the initial position $(23, 45, 17, 49)$. If the winning moves in this position exist, find all of them. Justify your answer.

Problem 4 of 5. Consider a partisan subtraction game with the subtraction set for the first player $S_1 = \{1,\ 3,\ 4\}$, and the subtraction set for the second player $S_2 = \{1,\ 2\}$. Find all the winning positions for the first player. Justify your answer.

Problem 5 of 5. The **Game of** Y is played on a triangular board tiled with hexagons. As in Hex, the two players take turns coloring in hexagons, each using his assigned color. A player wins when he establishes a Y, a monochromatic connected region in his color that meets all three sides of the triangle. Please look at Karlin-Peres book, figure 1.12, for a picture.

Prove that the first player has a winning strategy. You can use Theorem 1.2.8.