Mathematical Introduction to Game Theory

Assignment 8, due November 27

Problem 1 of 5. Let (N, v_1) and (N, v_2) be two games in coalitional form with non-empty cores. Prove that for any positive $\lambda > 0$ and $\mu > 0$, the function $v := \lambda v_1 + \mu v_2$ is a characteristic function and that the game (N, v) has a non-empty core.

Problem 2 of 5. Find the characteristic function of the 3-person game in strategic form when the payoff vectors are:

If I chooses 1:

$$\begin{pmatrix} (2,7,-2) & (3,0,1) \\ (-1,6,3) & (3,-2,1) \end{pmatrix}$$

If I chooses 2:

$$\begin{pmatrix} (-1,2,4) & (1,3,3) \\ (7,5,-4) & (3,-2,1) \end{pmatrix}$$

Problem 3 of 5. (**Oil Market game.**) Country 1 has oil which it can use to run its transport system at a profit of a per barrel. Country 2 wants to buy the oil to use in its manufacturing industry, where it gives a profit of b per barrel, while Country 3 wants it for food manufacturing where the profit is c per barrel. Let $a < b \leq c$.

(1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.

(2) Describe all the imputations.

(3) Compute the core of the game.

(4) Find the Shapley Value.

Problem 4 of 5. Prove that the game in coalitional form (N, v) is inessential if and only if $-v$ is a characteristic function.

Problem 5 of 5. A toy costs \$400 and consists of three parts: I, II, III. There is one manufacturer of part I, two manufacturers of part II, and three manufacturers of part III. No part of the toy can be sold separately.

(1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.

(2) Describe all the imputations.

(3) Compute the core of the game.

(4) Find the Shapley Value.