# DECEMBER 2019 PRACTICE FINAL EXAMINATION
# MAT406H5F
# Mathematical Introduction to Game Theory

**Problem 1 (20points)**. In a splitting game, the players are given a few non-empty piles of stones. A legal move consists of splitting a pile into two non-empty piles. The winner makes the last move. Thus the terminal positions are the positions which have no piles with more than one stone.

Compute the SG-function of the game and use it to determine which starting one-pile positions are winning for the first player to move. Justify your answer.

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**Problem 2 (20points)**. Solve (i.e. find the value of the game and optimal strategies for both players) the two-person zero-sum game given by the following matrix.

$$\begin{pmatrix} 0 & 1 & 1 & 2 & -1 \\ -1 & 2 & 0 & 0 & 2 \\ 1 & 1 & -2 & 2 & 0 \\ -2 & -1 & -1 & 1 & 1 \end{pmatrix}$$

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**Problem 3 (20points)**. Find the safety levels, all the corresponding max-min strategies, and all of the Nash Equilibria for the game given in the strategic form by the following bi-matrix.

$$\begin{pmatrix} (1,5) & (2,8) & (3,3) & (2,7) \\ (5,3) & (0,4) & (4,2) & (6,2) \\ (2,4) & (3,5) & (5,3) & (3,6) \end{pmatrix}$$

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**Problem 4 (20points)**. The game is given by the following bi-matrix.

$$\begin{pmatrix} (2,2) & (3,1) & (2,1) & (9,8) & (1,2) \\ (4,5) & (5,4) & (1,1) & (1,0) & (3,1) \\ (4,3) & (2,0) & (1,3) & (3,1) & (4,4) \\ (-1,1) & (5,5) & (5,6) & (1,0) & (-2,-3) \end{pmatrix}$$

(1) Solve the game as a TU game, i.e. find the point of optimal agreement.
**Hint:** You have already solved half of this problem during this test.

(2) Solve the game as an NTU game using the Nash model with the threat point $(0,0)$.
**Hint:** Recall that the Nash solution used axioms.

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**Problem 5 (20points)**. A pair of gloves costs \$20. A left glove or a right glove cannot be sold separately. Assume that there is one manufacturer of the right glove and four manufacturers of the left glove.

(1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
(2) Describe all the imputations.
(3) Compute the core of the game.
(4) Find the Shapley Value.

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