MAT137 - Week 7, lecture 2

- Reminder: Problem Set 3 is due next Thursday, November 1, at 11:59pm.
  - Don't leave the submission process until the last minute!
- In today's lecture we'll about the chain rule and trigonometric derivatives, and maybe implicit differentiation if there's time.
- For next Tuesday's lecture, watch all remaining videos on Playlist 3, starting from 3.12.
  - Video 3.14 is for those students who need a reminder of the basics of logarithms.
  - Videos 3.19 and 3.20 are examples of "related rates" problems, which we'll see next week.

Using the Chain Rule

Problem. Compute the derivatives of the following functions:

1. $f(x) = (3x^7 + 4x + 1)^{2018}$.

2. $g(x) = (x^2 + 1)^7(4x^3 + 2x)^{13}$.

Note, these are derivatives you technically could have computed before. The Chain Rule just makes things much easier.

Derivatives of compositions of functions

Problem. Assume f and g are functions that have all their derivatives. Find formulas for

1. $(f \circ g)'(x)$
2. $(f \circ g)''(x)$
3. $(f \circ g)'''(x)$

in terms of the values of f, g, and their derivatives.

Hint: The first one is simply the chain rule.

Chain Rule + Product Rule $\Longrightarrow$ Quotient Rule

Problem 1. Let $c \in \mathbb{R}$, and let g be a function which is differentiable at c and such that $g(x) \neq 0$ for all x near c.

Let $h(x) = \dfrac{1}{g(x)}$.

Use the chain rule to derive a formula for $h'(c)$.

Problem 2. Use your formula from above to give a simple proof of the quotient rule.

To this day, this is how I remember the quotient rule.

Derivative of cos

In one of the videos, you saw a derivation of the derivative of sin.

Let $g(x) = \cos x$.

Derive a formula for its derivative directly from the definition of the derivative as a limit.

Hint: Use the "$h \to 0$" version of the definition of the derivative, and imitate the derivation in Video 3.11.

Hint: This identity may come in handy:

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

Trigonometric derivatives

You now know that:

$$\frac{d}{dx}\sin x = \cos x \quad \text{and} \quad \frac{d}{dx}\cos x = -\sin x$$

Problem. Evaluate the following limits:

1. $\lim_{h\to 0} \dfrac{\cos(7(x+h)) - \cos(7x)}{h}$.

2. $\lim_{h\to 0} \dfrac{\sin(7x+h) - \sin(7x)}{h}$

Trigonometric derivatives

We didn't see this slide in class, but it's a simple exercise you should do for yourself.

Again, you now know that:

$$\frac{d}{dx}\sin x = \cos x \quad \text{and} \quad \frac{d}{dx}\cos x = -\sin x$$

From these two results, use the quotient rule to quickly derive the formulas for the derivatives of...

1. $\tan(x)$
2. $\sec(x)$
3. $\csc(x)$
4. $\cot(x)$

(In practise, this is what I do in my head every time for the latter three functions. I don't like remembering things.)