Welcome to MAT137!

Section L0201 – TRF10 in MP203

Instructor	Ivan Khatchatourian
Email	`ivan@math.toronto.edu` (**always** put "MAT137" in the subject line)
Office hours	Thursdays 11am-1pm, PG003, **starting next week** (**always** check the course website for office hours)

Welcome to MAT137!

- Course website: `http://uoft.me/MAT137`
- Make sure you have read and understood the course outline.
  (To find it, go to: Course website → Resources.)
- Make sure you check your UofT email regularly for announcements.
  (Google "utmail mobile" for info on how to set up phone/tablet access, or just go to this link.)
- Join Piazza, our online help forum.
  (For links, go to: Course website → Resources.)

Welcome to MAT137!

- My page for just our section.
  (To find it, go to: Course website $\rightarrow$ Resources and click on my name.)

  Please get into the habit of regularly checking both the course website and the page above.

  Our section's page will tell you which videos to watch before each lecture. **For tomorrow's lecture, watch videos 1.1 through 1.3.**

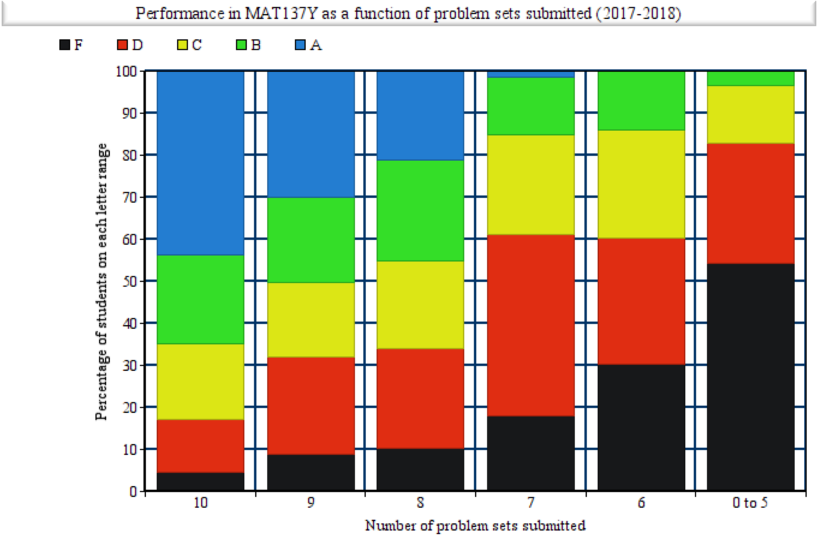
- Precalculus review: http://uoft.me/precalc

  (Strong precalc skills are the most important prerequisite of this course.)

  Going through this material is "Problem Set 0" (not to be graded).

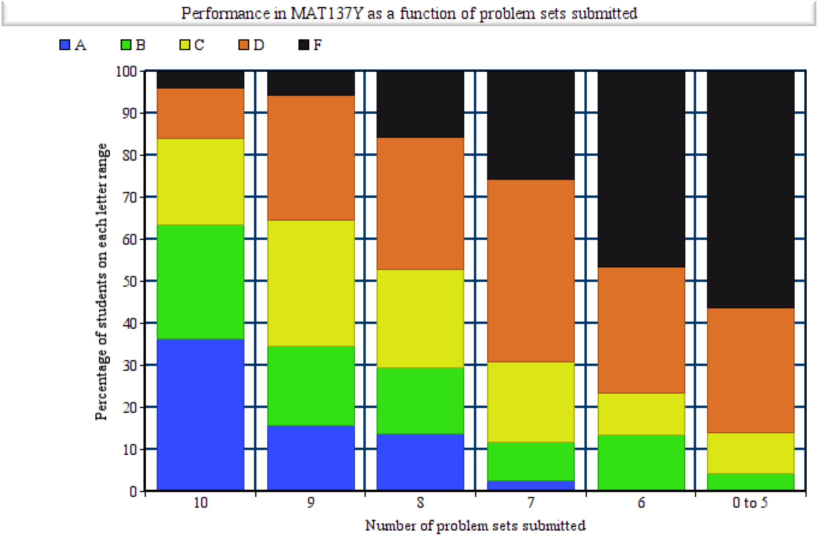
How did students do last year?

MAT137 results from 2017–2018:



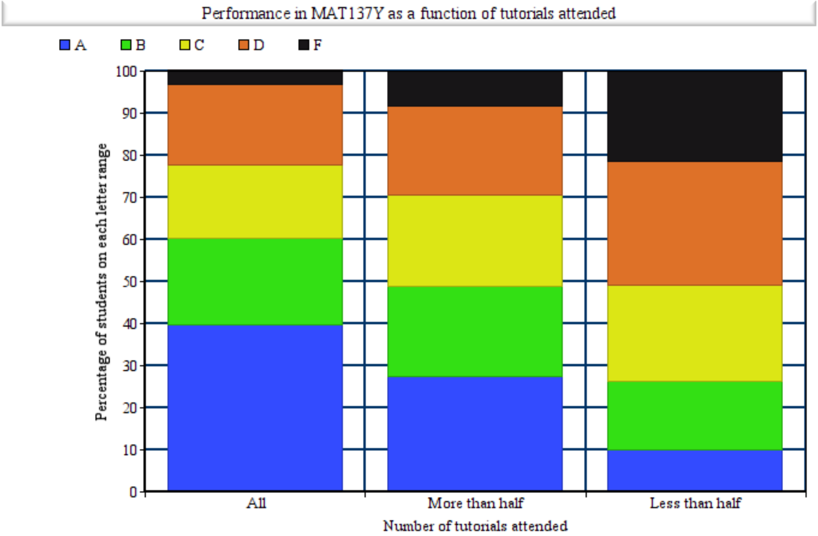
How did students do two years ago?

MAT137 results from 2016–2017:



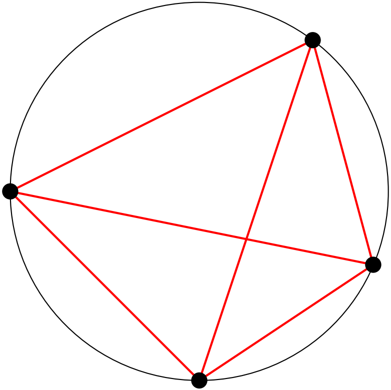
How did students do two years ago?

MAT137 results from 2016–2017:

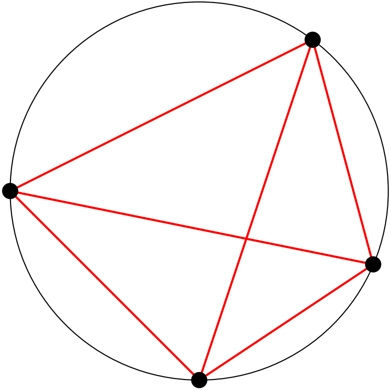


Proofs are important

- Pick 4 points at random on a circle (not necessarily evenly spaced).
- Join every pair of points.
- Into how many regions is the circle divided?

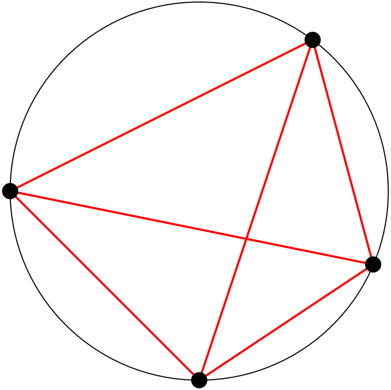


Proofs are important (part 2)



# of points	# of regions
1	
2	
3	
4	8
5	

Proofs are important (part 3)



# of points	# of regions
1	1
2	2
3	4
4	8
5	16
6	??
n	??

Actual formula: $\binom{n}{4} + \binom{n}{2} + 1 = \frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24)$.

(Proving this is not easy. Check out this Wikipedia article for an explanation, if you're interested.)

Another famous example

Consider the function

$$\pi(x) = \# \text{ of prime numbers less than or equal to } x.$$

For example:

$$\begin{array}{rr} \pi(2) = 1 & \pi(10) = 4 \\ \pi(3) = 2 & \pi(11) = 5 \\ \pi(4) = 2 & \pi(100) = 25 \end{array}$$

This function is *extremely important* to number theorists, but it is not very well understood. A much simpler function, called $\mathrm{li}(x)$ was proved to approximate $\pi(x)$ quite well in 1896.

Another famous example (part 2)

For all integers n that anyone has ever checked (even to this day), we have found that

$$\pi(n) - \mathrm{li}(n) < 0.$$

In other words, $\mathrm{li}(n)$ always seems to *overestimate* $\pi(n)$.

There is literally no numerical evidence that $\mathrm{li}(n)$ ever underestimates $\pi(n)$, even for a single value of n.

However, in 1914 J. E. Littlewood proved that $\pi(n) - \mathrm{li}(n)$ switches sign *infinitely many times* as n increases!

The earliest estimate (made in 1955) for the first place the sign becomes positive was on the order of $10^{10^{10^{964}}}$, which is an outrageously large number. We've since improved this bound to around 1.4×10^{316}.