MAT137 - Week 2, Lecture 3

- **Reminder:** Tutorials start next week. Make sure you are enrolled in one.
- **Reminder:** Problem Set 1 is available on the course website, and is due **Thursday, September 26 by 11:59pm**.
  - You will get an email about a week before it's due telling you how to submit it online.
- Today's lecture will assume you have watched up to and including video 1.13.

  For next Tuesday's lecture, watch videos 1.14 and 1.15.
- Homework exercise: Complete the proof of the theorem about even and odd numbers.

Evens and odds

Recall that last week we found a way of writing the set of even integers in set builder notation with quantifiers.

Let $x \in \mathbb{R}$. Which of the following is a correct definition for "x is odd"? If you think one of them is not a definition, give a reason why.

1. x is odd if $x = 2n + 1$.
2. x is odd if $\forall n \in \mathbb{Z}$, $x = 2n + 1$.
3. x is odd if $\exists n \in \mathbb{Z}$ such that $x = 2n + 1$.
4. x is odd if $\exists n \in \mathbb{Z}$ such that $x = 2n + 1$.

Having established the definition of oddness, evenness is similar enough to be easy:

$$\text{Let } x \in \mathbb{R}.\ x \text{ is even if } \exists n \in \mathbb{Z},\ x = 2n.$$

Evens and odds (part 2)

Consider the following theorem, which I hope you believe:

Theorem

The sum of any two odd integers is even.

Take a moment to think about how you would write a formal proof of this.

In particular, what would the structure of the proof be? How must a proof of this statement begin? How must it end?

Evens and odds (part 3)

Theorem

The sum of any two odd integers is even.

What are some (of the many, many) things wrong with the following "proof"?

Proof.

$x = 2a + 1$

$y = 2b + 1$

$x + y = 2n$

$(2a + 1) + (2b + 1) = 2n$

$2(a + b + 1) = 2n$

$a + b + 1 = n.$ □

Evens and odds (part 4)

Theorem

The sum of any two odd integers is even.

What about the following "proof":

Proof.

For all n:
EVEN + EVEN = EVEN
EVEN + ODD = ODD
ODD + ODD = EVEN □

I have actually seen someone write this. =(

Problem: Write a proof for this statement that is less awful.

Definitions - Injectivity

Let f be a function with domain D.

f is called injective on D (or sometimes one-to-one on D) if different inputs to the function always yield different outputs.

In other words, different values of x produce different values of $f(x)$.

For example, the function $f(x) = x$ is injective, while the function $g(x) = x^2$ is not.

Problem. Write a formal definition for this property.

Definitions - Injectivity (continued)

Let f be a function with domain D. Which of these is a definition of "f is injective on D"? For those that are not, what (if anything) *do* they mean?

1. $f(x_1) \neq f(x_2)$. $\leftarrow$ meaningless.
2. $\exists x_1, x_2 \in D$ such that $f(x_1) \neq f(x_2)$. $\leftarrow$ definition of "f is not constant on D".
3. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$. $\leftarrow$ meaningless, more or less.
4. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.
5. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$.
6. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$.
7. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.