MAT137 - Week 3, Lecture 1

- **Reminder:** Problem Set 1 is available on the course website, and is due **Thursday, September 26 by 11:59pm**.
  - You will get an email about a week before it's due telling you how to submit it online.
- Today's lecture will assume you have watched all of playlist 1.

  For this Thursday's lecture, watch video 2.4.

Evens and odds

Last class we discussed this theorem, after stating definitions of evenness and oddness.

Theorem

The sum of any two odd integers is even.

We also talked about the many problems with this "proof":

Proof.

$x = 2a + 1$
$y = 2b + 1$
$x + y = 2n$
$(2a + 1) + (2b + 1) = 2n$
$2(a + b + 1) = 2n$
$a + b + 1 = n.$ $\square$

Evens and odds (part 2)

Theorem

The sum of any two odd integers is even.

Problem. Write a proof for this statement that is less awful.

Evens and odds (part 3)

Theorem

The sum of two odd integers is even.

Here's how I might write a proof of this fact:

Proof.

Let x and y be two odd integers. By the definition of oddness, there must exist two integers n and m such that

$$x = 2n + 1 \quad \text{and} \quad y = 2m + 1.$$

Then we can compute:

$$x + y = (2n + 1) + (2m + 1) = 2n + 2m + 2 = 2(n + m + 1).$$

We know $2(n + m + 1)$ is even by the defininition of evenness, and therefore $x + y$ is even. $\square$

Definitions - Injectivity

Let f be a function with domain D.

f is called <u>injective on D</u> (or sometimes <u>one-to-one on D</u>) if different inputs to the function always yield different outputs.

In other words, different values of x produce different values of $f(x)$.

For example, the function $f(x) = x$ is injective, while the function $g(x) = x^2$ is not.

Problem. Write a formal definition for this property.

Definitions - Injectivity (continued)

Let f be a function with domain D. Which of these is a definition of "f is injective on D"? For those that are not, what (if anything) *do* they mean?

1. $f(x_1) \neq f(x_2)$. $\leftarrow$ meaningless.
2. $\exists x_1, x_2 \in D$ such that $f(x_1) \neq f(x_2)$. $\leftarrow$ definition of "f is not constant on D".
3. $\forall x_1, x_2 \in D$, $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$. $\leftarrow$ meaningless, more or less.
4. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$. $\leftarrow$ every function satisfies this.
5. $\forall x_1, x_2 \in D$, $f(x_1) \neq f(x_2) \implies x_1 \neq x_2$. $\leftarrow$ definition of "f is a function".
6. $\forall x_1, x_2 \in D$, $f(x_1) = f(x_2) \implies x_1 = x_2$.
7. $\forall x_1, x_2 \in D$, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.