MAT137 - Week 3, Lecture 2

- **Reminder:** Problem Set 1 is available on the course website, and is due **Thursday, September 26 by 11:59pm**.
  - You will get an email about a week before it's due telling you how to submit it online.
- Today's lecture will assume you have watched all of playlist 1, and video 2.4.

  For tomorrow's lecture, watch video 2.1 through 2.3.

Injectivity

Last class, we settled on two equivalent definitions of injectivity:

Definition

Let f be a function with domain D. f is injective on D if either of the following equivalent statements is true.

- $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$.
- $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

A simple theorem about injectivity

In one of the videos, you learned the definition of an increasing function. Here it is again, for reference:

Definition

A function f is increasing on an interval D if

$$\forall x_1, x_2 \in D,\ x_1 < x_2 \implies f(x_1) < f(x_2)$$

Write a formal proof for the following theorem.

Theorem

Let f be a function defined on an interval D.

If f is increasing on D, then f is injective on D.

Sample proof of the theorem

Proof.

Let f be a function defined on an interval D. Assume f is increasing on D.

We want to show that $\forall x_1, x_2 \in D$, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

Fix arbitrary points $x_1, x_2 \in D$. Assume $x_1 \neq x_2$.

It must be that either $x_1 < x_2$ or $x_2 < x_1$. Suppose we're in the first case. Then since f is increasing on D, we know that $f(x_1) < f(x_2)$. In particular, this means $f(x_1) \neq f(x_2)$, which is what we needed to show.

The case in which $x_2 < x_1$ is exactly analogous. □

Absolute values and inequalities

Problem 1. For which values of x is the following inequality true?

$$|x - 7| < 3$$

In other words, which values of x are within a distance 3 from 7?

Notice that thinking about the expression $|x - 7|$ as "the distance between x and 7" makes this problem *much* easier.

What about the following inequality?

$$0 < |x - 7| < 3$$

Absolute values and inequalities (part 2)

Problem 2. Suppose x is a real number that satisfies the inequality

$$|x-2| < 1.$$

What bounds, if any, can you put on $|x-7|$?

Here's how you should read this question:

If x is within a distance 1 from 2, how far can x be from 7?

Again, I hope you agree that when phrased in terms of distances, this is a straightforward problem.

You have just proved the following conditional:

$$|x-2| < 1 \implies 4 < |x-7| < 6$$

Absolute values and inequalities (part 3)

Now we know how to bound values of x with inqualities and absolute values. Let's use bounds on x to find bounds on the values of *functions* of x.

Problem 3. Suppose x is a real number that satisfies the inequality

$$|x+7| < 2.$$

How big can $|3x+21|$ be?

In words:

If x is within a distance 2 from -7, how far can $3x$ be from -21?

You have just proved the following conditional:

$$|x+7| < 2 \implies |3x+21| < 6$$

A Greek letter!

Learn to write a lowercase Greek letter *delta*:

$$\delta$$