MAT137 - Week 3, Lecture 3

- **Reminder:** Problem Set 1 is available on the course website, and is due **Thursday, September 26 by 11:59pm**.
  - You will get an email about a week before it's due telling you how to submit it online.
- **Reminder:** Starting next week, our Tuesday lectures will be in MP202 (next door). Thursday and Friday lectures remain in MP203.
- Today's lecture will assume you have watched up to and including video 2.4.

  For next Tuesday's lecture, watch video 2.5 and 2.6.
- You have a homework assignment from this lecture. See the last slide.

Absolute values and inequalities

Last class, we started a sequence of problems about absolute values, inequalities, and conditional statements.

Problem 1. For which values of x is the following inequality true?

$$|x - 7| < 3$$

(Answer: $x \in (4, 10)$)

Problem 2. You proved this conditional statement:

$$|x - 2| < 1 \implies 4 < |x - 7| < 6$$

Problem 3. You proved this conditional statement:

$$|x + 7| < 2 \implies |3x + 21| < 6$$

A Greek letter!

Learn to write a lowercase Greek letter *delta*:

$$\delta$$

Absolute values and inequalities (part 2)

Now let's reverse idea in the previous exercise.

Problem 4. Find **one** positive number δ that makes the following conditional true.

$$\text{If } |x-3| < \delta, \text{ then } |4x-12| < 6.$$

Problem 5. Find **all** positive numbers δ that make the above conditional true.

Absolute values and inequalities (part 3)

Now we know that for any $0 < \delta \leq \frac{3}{2}$, the following conditional is true:

$$\text{If } |x-3| < \delta, \text{ then } |4x-12| < 6.$$

We'll work with this idea a bit.

Problem 6. Suppose we want a tighter restriction on $|4x-12|$ in the conditional above. For example, let's say we want the distance between $4x$ and 12 to be less than 1:

$$\text{If } |x-3| < \delta, \text{ then } |4x-12| < 1.$$

Will all of the same values of δ that worked before work now?

No! To make $4x$ closer to 12, we must make x closer to 3. Which values of δ will work here?

Another Greek letter!

Learn to write a lowercase Greek letter *epsilon*:

$$\varepsilon$$

(It's essentially a backwards number "3". Yours doesn't have to have the little loop.)

Absolute values and conditionals (part 4)

Now, let's generalize. Why solve just one problem when we can solve *all* the problems?

Problem 7. Let ε be a fixed positive real number. Is it possible to find a positive δ that **does not** depend on ε and that makes the following conditional true?

$$\text{If } |x-3| < \delta, \text{ then } |4x-12| < \varepsilon.$$

No! The smaller ε is, the smaller δ should have to be! Just like before.

Problem 8. Let ε be a fixed positive real number. Find a value of δ, in terms of ε, that makes the following conditional true:

$$\text{If } |x-3| < \delta, \text{ then } |4x-12| < \varepsilon.$$

Absolute values and conditionals (part 5)

Now we know that for a fixed positive real number ε, the following conditional is true:

$$\text{If } |x-3| < \frac{\varepsilon}{4}, \text{ then } |4x-12| < \varepsilon.$$

Congratulations! You've just done the "hard part" of proving that

$$\lim_{x\to 3} 4x = 12.$$

Not so bad, right?

Limits, intuitively

Recall the intuitive definition of a limit given in the videos:

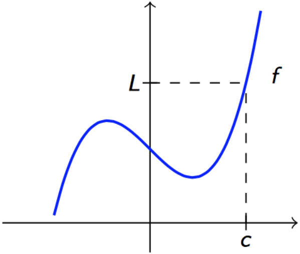
$$\lim_{x \to c} f(x) = L \qquad \text{means}$$

If x is close to c (but not equal to c), then $f(x)$ is close to L.

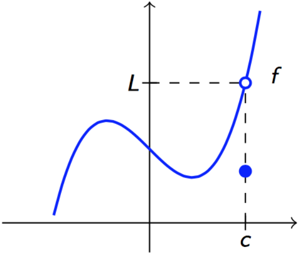
Limits, intuitively

Note that a limit ***never*** cares about what's happening at c. Only near c.

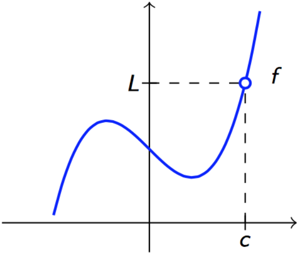
All of the following functions have the same limit L at c:



$L = f(c)$



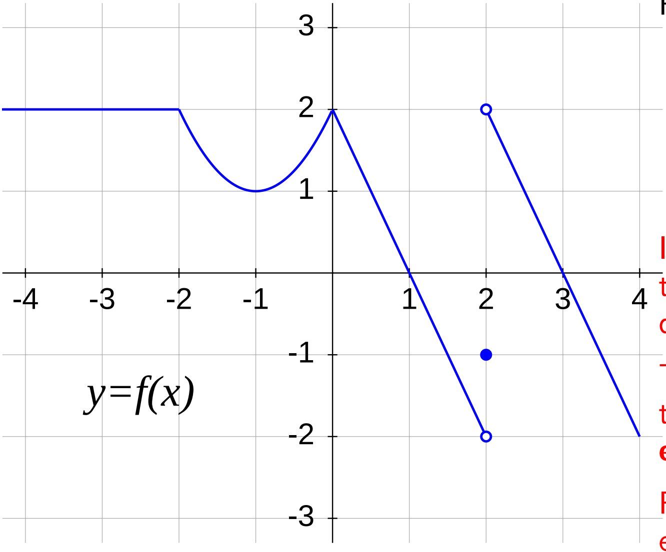
$L \neq f(c)$



f is not defined at c

In particular, this means that in principle you can ***never*** evaluate a limit simply by plugging $x = c$ into the function.

Limits from a graph



Find the value of:

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0} f(f(x))$
3. $\lim_{x \to -3} f(f(x))$

In class, we agreed that the first one doesn't exist.

Then I told you that the other two **do exist**.

For homework, evaluate them.