MAT137 - Week 4, Lecture 2

- **Reminder:** Problem Set 1 is available on the course website, and is due **Thursday, September 26 by 11:59pm**.
  - You will get an email about a week before it's due telling you how to submit it online.
- Today's lecture will assume you have watched up to and including video 2.9.

  For tomorrow's lecture, watch videos 2.10 and 2.11.

Limits of the floor function

Given a real number x, we define the floor of x, denoted by $\lfloor x \rfloor$, to be the largest integer smaller than or equal to x. For example:

$$\lfloor \pi \rfloor = 3, \qquad \lfloor 7 \rfloor = 7, \qquad \lfloor -0.5 \rfloor = -1.$$

Sketch the graph of $y = \lfloor x \rfloor$. Then compute:

1. $\lim_{x \to 0^+} \lfloor x \rfloor$
2. $\lim_{x \to 0^-} \lfloor x \rfloor$
3. $\lim_{x \to 0} \lfloor x \rfloor$
4. $\lim_{x \to 0} \lfloor x^2 \rfloor$

Defining side limits

Last class we wrote down the following definition.

Definition

Let $a, L \in \mathbb{R}$. Let f be a function defined at least on an interval centred at a, except possibly at a.

Then $\lim_{x \to a} f(x) = L$ means

$$\forall \varepsilon > 0 \; \exists \delta > 0 \text{ such that } 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Now, write formal definitions of:

$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

Defining infinite limits

I didn't spend time on this in class. Think about it for homework.

In video 2.6, you saw the definition of $\lim_{x \to \infty} f(x) = L$.

Let's do a similar new thing now.

Let $a \in \mathbb{R}$. Let f be a function defined at least on an interval centred at a, except possibly at a.

Write a formal definition for

$$\lim_{x \to a} f(x) = \infty.$$

Warm up for your first $\varepsilon - \delta$ proof.

1. Find **one** positive value of δ such that
$$|x-3| < \delta \implies |7x-21| < 1.$$
2. Find **all** positive values of δ such that
$$|x-3| < \delta \implies |7x-21| < 1.$$
3. Find **all** positive values of δ such that
$$|x-3| < \delta \implies |7x-21| < \frac{1}{100}.$$
4. Let ε be an arbitrary positive number. Find **all** positive values of δ such that
$$|x-3| < \delta \implies |7x-21| < \varepsilon$$

Your first $\varepsilon - \delta$ proof.

Come to next class with a written proof for this problem.

Problem. Prove, directly from the formal definition of the limit, that

$$\lim_{x \to 3} (7x - 15) = 6.$$

Follow these steps, in order:

1. Write down the formal definition of what you're trying to prove. Without the definition, you can't prove anything.
2. Write down what the structure of the proof should be, without filling in any details. What variables must you define in what order, what must you assume and where, etc. This part should require little to no thinking.
3. Do some rough work to figure out what value of δ will work.
4. Finally, write the complete proof.

A sample proof. What's wrong with it?

Proof.

$$\begin{aligned} |(7x-15)-6| &< \varepsilon \\ |7x-21| &< \varepsilon \\ 7|x-3| &< \varepsilon \\ |x-3| &< \frac{\varepsilon}{7} \end{aligned}$$

So $\delta = \dfrac{\varepsilon}{7}$. $\square$

(Nearly everything is wrong with this proof.)

This is an example of what the rough work you do *before writing the proof* might look like.