MAT137 - Week 5, Lecture 1

- **Reminder:** Problem Set 2 is available on the course website, and is due **Thursday, 10 October by 11:59pm**.
- Today's lecture will assume you have watched up to and including video 2.13.

  For Thursday's lecture, watch videos 2.14 and 2.15. We probably won't actually get to talk about this subject next class though.
- Finish the proof on the last slide for next class.

Finishing a proof from last class.

Goal. Prove, directly from the formal definition of the limit, that

$$\lim_{x \to 0}(x^3 + x^2) = 0.$$

Last class you wrote the definition of this statement, so you know what you have to prove.

Finishing a proof from last class (part 2).

Last class, we proved that if ε and C are positive real numbers, then:

$$|x| < \min\left\{\sqrt{\frac{\varepsilon}{C}}, 6\right\} \implies \begin{cases} Cx^2 < \varepsilon \\ |x+1| < 7 \end{cases}$$

Use the fact above to solve this problem:

Problem 1. Find **one** value of $\delta > 0$ such that $|x| < \delta \implies |x^3 + x^2| < \varepsilon$.

Using your solution to Problem 1, complete the proof of the original limit:

Problem 2. Prove, directly from the formal definition of the limit, that

$$\lim_{x \to 0} (x^3 + x^2) = 0.$$

Proving a limit does not exist.

Goal.

We want to prove, directly from the definition, that

$$\lim_{x \to 0^+} \frac{1}{x} \text{ does not exist.}$$

First, write down the formal definition of what you're trying to prove. You saw the definition of a regular limit not existing in video 2.9, and the corresponding definition for a one-sided limit has the same structure.

Second, write down what the structure of the proof should be, without filling in any details. What variables must you define in what order, what must you assume and where, etc.

Third, write the complete proof.