MAT137 - Week 5, Lecture 2

- **Reminder:** Problem Set 2 is available on the course website, and is due **Thursday, 10 October by 11:59pm**.
- Today's lecture will assume you have watched up to and including video 2.13.

  (I am now accepting the fact that we're behind schedule, and adjusting what I'm asking you to watch.)

  For tomorrow's lecture, watch videos 2.14 and 2.15.

Proving a limit does not exist.

Goal.

We want to prove, directly from the definition, that

$$\lim_{x \to 0^+} \frac{1}{x} \text{ does not exist.}$$

Last class, we agreed that the statement we must prove here is:

$$\forall L \in \mathbb{R}, \quad \lim_{x \to 0^+} \frac{1}{x} \neq L.$$

First, write the formal definition of this statement.

Second, write down what the structure of the proof should be, without filling in any details. What variables must you define in what order, what must you assume and where, etc.

Third, write the complete proof.

A simple theorem about limits.

We didn't see this in class. I'm posting it as a nice practise problem.

Goal.

Let f be a function with domain $\mathbb{R}$, and suppose that

$$\lim_{x \to 0} f(x) = 7.$$

Prove, directly from the formal definition of the limit, that

$$\lim_{x \to 0} \left[2f(5x) \right] = 14.$$

(Do not use any limit laws.)

1. Write the formal definition of the statement you are trying to prove.
2. Write down what the structure of the proof should be.
3. Do a bunch of rough work.
4. Write the complete proof.

Critiquing your proofs

(Continuation of exercise on previous slide.)

After your proofs are written, read them and ask the following questions:

1. Is the structure of the proof correct? (e.g., does it begin by fixing $\varepsilon > 0$, then defining δ, etc.)
2. Does the proof clearly state what δ is?
3. Is the proof self-contained? In other words, do you need to read the rough work in order to understand it?
4. Are all of the variables defined? Are they defined in the correct order?
5. Does each step follow logically from the previous steps? Is that logic explained?
6. Does it actually prove what it was supposed to prove?
7. Does the proof assume the conclusion at some point?

Defining infinite limits

Which of these is a correct definition of $\lim_{x \to a} f(x) = \infty$?

1. $\forall M \in \mathbb{R},\ \exists \delta > 0$ such that $0 < |x - a| < \delta \implies f(x) > M$.
2. $\forall M > 0,\ \exists \delta > 0$ such that $0 < |x - a| < \delta \implies f(x) > M$.
3. $\forall M > 27,\ \exists \delta > 0$ such that $0 < |x - a| < \delta \implies f(x) > M$.
4. $\forall M \in \mathbb{N},\ \exists \delta > 0$ such that $0 < |x - a| < \delta \implies f(x) > M$.
5. $\forall M \in \mathbb{R},\ \exists \delta > 0$ such that $0 < |x - a| < \delta \implies f(x) \geq M$.

Make sure to think about this with pictures.