MAT137 - Week 6, Lecture 1

- **Reminder:** Problem Set 2 is available on the course website, and is due **Thursday, 10 October by 11:59pm**.
- Today's lecture will assume you have watched up to and including video 2.18.

  For Thursday's lecture, watch videos 2.19 through 2.22.

Warm-up: Did you watch the videos?

Let $a \in \mathbb{R}$, and let f and g be functions that are defined on all of $\mathbb{R}$.

Assume that $\lim_{x \to a} g(x)$ exists and equals a real number L.

Is it necessarily true that

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(L)?$$

No! We need f to be continuous at L.

Come up with a counterexample. (You've already seen some throughout the course.)

The theorem from the videos about this:

Theorem

Let $a \in \mathbb{R}$, and let f and g be functions such that:

- $\lim_{x \to a} g(x) = L$.
- *f is continuous at L.*

Then,

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(L).$$

(Proving this theorem is one the problems for Playlist 2. I suggest doing it!)

Limits and compositions (part 2)

Problem. Find functions f and g, defined on all of $\mathbb{R}$, such that:

- f and g are both continuous at 0.
- $f(g(x))$ is not continuous at $x = 0$.

Does this contradict the theorem we just saw? Why or why not?

Trigonometric limit computations

Using the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (and maybe some simple trig identities), compute the following limits:

1. $\lim_{x\to 2} \frac{\sin x}{x}$
2. $\lim_{x\to 0} \frac{\sin(5x)}{x}$
3. $\lim_{x\to 0} \frac{\tan^2(2x^2)}{x^4}$
4. $\lim_{x\to 0} \frac{\sin e^x}{e^x}$
5. $\lim_{x\to 0} \frac{1-\cos x}{x}$
6. $\lim_{x\to 0} \frac{\tan^{10}(2x^{20})}{\sin^{200}(3x)}$
7. $\lim_{x\to 0} [(\sin x)(\cos(2x))(\tan(3x))(\sec(4x))(\csc(5x))(\cot(6x))]$

   I didn't show you this last one in class, but think about it before next class.