MAT137 - Week 6, Lecture 2

- **Reminder:** Problem Set 2 is due **today, by 11:59pm**.
- Problem Set A is on the website now. It contains material that is not covered by Problem Sets 1 and 2, but that is covered by Test 1. It is not to be submitted, but it is very good practise. Do these problems before Test 1.
- Today's lecture will assume you have watched up to and including video 2.22.

  For tomorrow's lecture, watch videos 3.1 through 3.3.
- Ivan wasn't able to make it to this lecture, so it was very kindly covered by Asif Zaman.

Trigonometric limit computations

Using the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (and maybe some simple trig identities), compute the following limits:

1. $\lim_{x\to 2} \frac{\sin x}{x}$
2. $\lim_{x\to 0} \frac{\sin(5x)}{x}$
3. $\lim_{x\to 0} \frac{\tan^2(2x^2)}{x^4}$
4. $\lim_{x\to 0} \frac{\sin e^x}{e^x}$
5. $\lim_{x\to 0} \frac{1-\cos x}{x}$
6. $\lim_{x\to 0} \frac{\tan^{10}(2x^{20})}{\sin^{200}(3x)}$
7. $\lim_{x\to 0} [(\sin x)\,(\cos(2x))\,(\tan(3x))\,(\sec(4x))\,(\csc(5x))\,(\cot(6x))]$

A question from a past test

Suppose the only thing we know about the function g is that

$$\lim_{x \to 0} \frac{g(x)}{x^2} = 2.$$

Compute the following limits (or explain why they do not exist):

1. $\lim_{x \to 0} \frac{g(x)}{x}$
2. $\lim_{x \to 0} \frac{g(x)}{x^4}$
3. $\lim_{x \to 0} \frac{g(3x)}{x^2}$

Limits at infinity

Compute the following limits.

1. $\lim_{x\to\infty} \left(x^7 - 2x^5 + 11\right)$
2. $\lim_{x\to\infty} \left(x^2 - \sqrt{x^5 + 1}\right)$
3. $\lim_{x\to\infty} \dfrac{x^2 + 11}{x + 1}$
4. $\lim_{x\to\infty} \dfrac{x^2 + 2x + 3}{3x^2 + 4x + 5}$
5. $\lim_{x\to\infty} \dfrac{x^3 + \sqrt{2x^6 + 1}}{2x^3 + \sqrt{x^5 + 1}}$

Which solution is correct?

Compute $L = \lim_{x \to -\infty} \left[x - \sqrt{x^2 + x} \right]$.

- **Solution 1**

$$L = \lim_{x \to -\infty} \frac{\left[x - \sqrt{x^2 + x} \right] \left[x + \sqrt{x^2 + x} \right]}{\left[x + \sqrt{x^2 + x} \right]} = \lim_{x \to -\infty} \frac{x^2 - (x^2 + x)}{\left[x + \sqrt{x^2 + x} \right]}$$

$$= \lim_{x \to -\infty} \frac{-x}{x \left[1 + \sqrt{1 + \frac{1}{x}} \right]} = \lim_{x \to -\infty} \frac{-1}{\left[1 + \sqrt{1 + \frac{1}{x}} \right]} = \frac{-1}{2}$$

- **Solution 2**

$$L = \lim_{x \to -\infty} \left[x - \sqrt{x^2 + x} \right] = (-\infty) - \infty = -\infty$$

Using the IVT and the EVT

Problem 1. Prove that the equation $x^3 - e^x - \cos(12x) + 100 = 0$ has at least two solutions