MAT137 - Week 6, Lecture 3

- **Reminder:** Problem Set A is on the website now. It contains material that is not covered by Problem Sets 1 and 2, but that is covered by Test 1. It is not to be submitted, but it is very good practise. Do these problems before Test 1.

- Today's lecture will assume you have watched up to and including video 3.3.

  For Tuesday's lecture, watch videos 3.4, 3.5, and 3.8.

- Ivan wasn't able to make it to this lecture, so it was very kindly covered by Asif Zaman.

Using the IVT and the EVT

Problem 1. Prove that the equation $x^3 - e^x - \cos(12x) + 100 = 0$ has at least two solutions

Problem 2. Prove that you were 3 feet tall at some point in your life.

Problem 3. Does the EVT imply that you will attain a maximum height?

Problem 4. Suppose that at half time during a Raptors basketball game, the Raptors have 51 points.

Is it necessarily true that at some point during the first half, they had exactly 20 points?

Problem 5. Prove that some point in Ivan's life, his height in inches equalled his weight in pounds.

Some data about Ivan:

- Height at birth: 20 inches.
- Weight at birth: 8 pounds.
- Height now: about 5 feet 9 inches (i.e., 69 inches).
- Weight now: None of your business. Guess!

Definition of a minimum

You didn't see this exercise in class, but it's a good problem to practise your understanding of quantifiers.

Problem. Let f be a function with domain D. Which of the following statements, if any, is a definition of

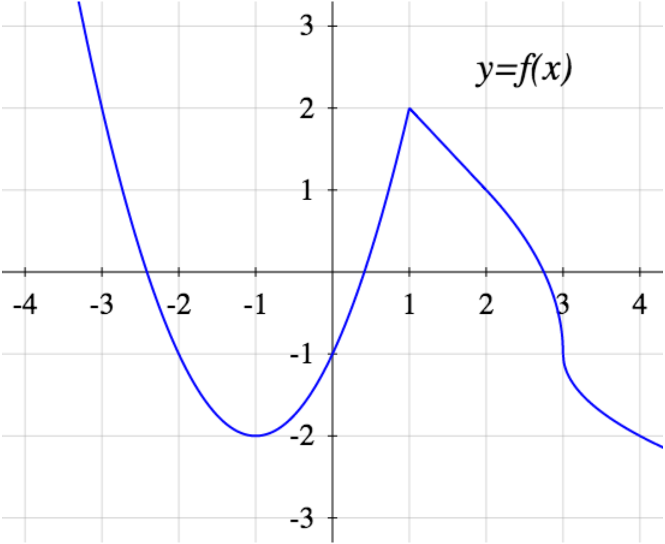
$$f \text{ has a minimum on } D.$$

If you think one of them is not a correct definition, find a counterexample.

1. $\forall x \in D$, $\exists C \in \mathbb{R}$ such that $f(x) \geq C$.
2. $\exists C \in D$ such that $\forall x \in D$, $f(x) \geq C$.
3. $\exists C \in \mathbb{R}$ such that $\forall x \in D$, $f(x) \geq C$.
4. $\exists C \in \mathbb{R}$ such that $\forall x \in D$, $f(x) > C$.
5. $\exists c \in D$ such that $\forall x \in D$, $f(x) \geq f(c)$.
6. $\exists c \in D$ such that $\forall x \in D$, $f(x) > f(c)$.

Derivatives from a graph

Below is the graph of a function f. Sketch the graph of its derivative f'.



Shapes of graphs

For each of these two graphs:

1. Sketch the graph of a continuous function that goes through the origin and whose derivative looks like the graph.
2. Sketch the graph of a non-continuous function whose derivative looks like the graph.