MAT137 - Week 7, Lecture 1

- **Reminder:** Test 1 will take place **this Friday**. See the course website for details.

- **Reminder:** Problem Set A is on the website now. It contains material that is not covered by Problem Sets 1 and 2, but that is covered by Test 1. It is not to be submitted, but it is very good practise. Do these problems before Test 1.

- Today's lecture will assume you have watched up to and including video 3.5, and video 3.8.

  For Thursday's lecture, watch videos 3.6, 3.7, and 3.9.

Some quick problems, to make sure you watched the videos

Problem 1. Let $c \in \mathbb{R}$, and suppose f is a function defined everywhere except at c. Which of the following must be true?

1. f is differentiable at c.
2. f cannot be differentiable at c.
3. We cannot say. f may or may not be differentiable at c.

Problem 2. Let $c \in \mathbb{R}$, and suppose f is differentiable everywhere except at c. Also suppose that f' is continuous everywhere except at c. Which of the following must be true?

1. f is continuous at c.
2. f cannot be continuous at c.
3. We cannot say. f may or may not be continuous at c.

Problem 3. Give an example of a function that is continuous everywhere, but whose derivative is not continuous at exactly one point.

People often misunderstand what a "tangent line" is.

I skipped over this problem in class, but you should think about it on your own if you're not clear on what a "tangent line" is.

Let C be a curve. Let P be a point on C.

Prove that each of these statements is false with a counterexample.

1. The line tangent to C at P intersects C at only one point: P.
2. If a line intersects C only at P,
   then that line must be the tangent line to C at P.
3. The tangent line to C at P intersects C at P and "does not cross" C at P. (This means that, near P, it stays on one side of C.)
4. If a line intersects C at P and "does not cross" C at P,
   then it is the tangent line to C at P.

Derivatives.

Problem 1. Let f be the function defined by $f(x) = x|x|$.

Is f differentiable at 0? If so, what is its derivative?

Hint: Write f as a piecewise function.

Another way to write this function is $f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$

When reviewing this problem, remember that our focus was on the justification for the correct answer.

Higher derivatives

Problem. Let $f(x) = \dfrac{1}{x^7}$.

1. Calculate the first few derivatives of f.

2. Make a conjecture for a formula for the n^{th} derivative of f.

3. Prove your formula. (The easiest way to do this is by induction.)