MAT137 - Week 7, Lecture 3

- **Reminder:** Test 1 will take place **today**. See the course website for details.

- Today's lecture will assume you have watched up to and including video 3.10.

  For next Tuesday's lecture, watch videos 3.11 and 3.12.

Warm-up: Using the Chain Rule

Problem. Compute the derivatives of the following functions:

1. $f(x) = (3x^7 + 4x + 1)^{2019}$.

2. $g(x) = (x^2 + 1)^7 \, (4x^3 + 2x)^{13}$.

Note, these are derivatives you technically could have computed before.

The Chain Rule just makes things much easier.

Derivatives of compositions of functions

Problem. Assume f and g are functions that have all their derivatives.

Find formulas for

1. $(f \circ g)'(x)$
2. $(f \circ g)''(x)$
3. $(f \circ g)'''(x)$

in terms of the values of f, g, and their derivatives.

Hint: The first one is simply the chain rule.

Chain Rule + Product Rule $\implies$ Quotient Rule

Problem 1. Let $c \in \mathbb{R}$, and let g be a function that is differentiable at c and such that $g(x) \neq 0$ for all x near c.

Let $h(x) = \dfrac{1}{g(x)}$.

Use the chain rule to derive a formula for $h'(c)$.

Problem 2. Use your formula from above to give a simple proof of the quotient rule.

(To this day, this is how I remember the quotient rule.)

An interesting function

Define a function h by

$$h(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

1. Is h continuous at 0?
2. Compute $h'(x)$ for any $x \neq 0$.
3. Using the definition of the derivative as a limit , determine whether h is differentiable at 0.
4. If you found that h is differentiable at 0, what is $h'(0)$?
5. Is h' continuous at 0?

Hint: The Squeeze Theorem will be useful to you here.