MAT137 - Week 8, Lecture 2

- **Reminder:** Problem Set 3 is available on the course website, and is due **Thursday, October 31 by 11:59pm**.
  - Don't leave the submission process until the last minute.

- Today's lecture will assume you have watched up to and including video 3.18.

  For tomorrow's lecture, watch videos 3.19 and 3.20.

Implicit differentiation exercises

Problem 1. The equation

$$\sin(x+y) + xy^2 = 0$$

defines a function $y = h(x)$ near $(0, 0)$.

Using implicit differentiation, compute

1. $h(0)$
2. $h'(0)$
3. $h''(0)$
4. $h'''(0)$

Problem 2. For the horrible curve from earlier:

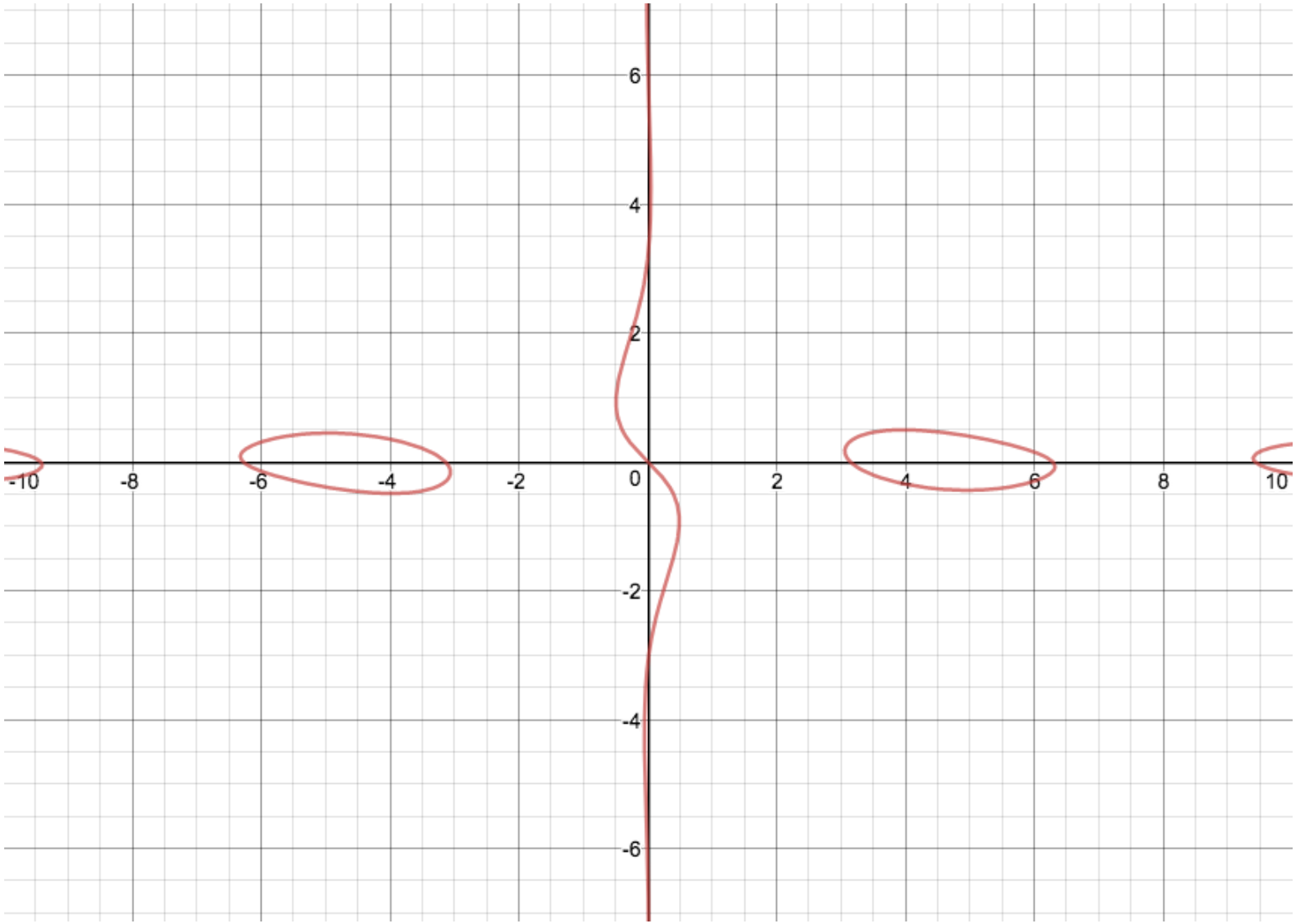
$$\tan^2(xy) = 3x^2y + \cos(y^2)$$

...try to compute y'.

Implicit differentiation exercises (part 2)

Here's what the first curve on the last slide looks like.

Explore this graph here: ▸ graph



Some derivatives with exponentials and logarithms

Problem. Compute the derivatives of the following functions:

1. $f(x) = e^{\sin x + \cos x} \ln(x)$
2. $f(x) = \pi^{\tan x}$
3. $f(x) = \ln\left[e^x + \ln(\ln(\ln(x)))\right]$

Reminder: We know:

- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} a^x = a^x \ln a$
- $\frac{d}{dx} \ln x = \frac{1}{x}$

True or false?

Problem. Let $f(x) = (x+1)^x$. Is the following formula true?

$$f'(x) = x \cdot (x+1)^{x-1}$$

False! This formula is trying to use the power rule for a situation it can't be used for.

The power rule only applies to functions of the form $g(x) = x^{\text{constant}}$.

Logarithmic differentiation to the rescue! If we take log of both sides, we get:

$$\ln(f(x)) = \ln\left((x+1)^x\right) = x\,\ln(x+1),$$

which we can now differentiate implicitly and isolate for f'.

Logarithmic differentiation.

Problem 1. Compute the derivatives of the following functions:

1. $f(x) = (x+1)^x$.
2. $g(x) = x^{\tan(x)}$.

Problem 2. Now generalize this technique into a new differentiation rule:

Let f and g be differentiable functions, and define h by

$$h(x) = [f(x)]^{g(x)}.$$

Derive a formula for $h'(x)$.