MAT137 - Week 9, Lecture 2

- **Reminder:** Problem Set 3 is available on the course website, and is due **today, by 11:59pm**.
  - Don't leave the submission process until the last minute.
- Today's lecture will assume you have watched up to and including video 4.5.

  For tomorrow's lecture, watch videos 4.6 and 4.8.
- This lecture ended a little abruptly in the middle of an exercise. That's my fault, and I apologise. Please finish that proof (on the last slide) at home, and we'll talk about it in class tomorrow.

Problem 1. Let f be a function with domain D. Write the definition of

$$f \text{ is injective on } D.$$

Problem 2. What is the relationship between injectivity and the existence of inverses?

Absolute values and inverses

Let h be the function defined by

$$h(x) = x|x| + 1.$$

1. Sketch the graph of h.
2. What are the domain, codomain, and range of h?
3. Does h have an inverse?
4. Compute $h^{-1}(-8)$.
5. Find an equation for $h^{-1}(x)$.
6. Use your equation to verify that:
   - for all $x \in \mathbb{R}$, $h(h^{-1}(x)) = x$
   - for all $x \in \mathbb{R}$, $h^{-1}(h(x)) = x$

Composition of injective functions

Assume that all functions in this problem have domain $\mathbb{R}$.

Prove the following theorem:

Theorem

Let f and g be functions.

IF f and g are injective, THEN $f \circ g$ is injective.

How to proceed:

1. Write the definition of what you want to prove.
2. Figure out the structure of the proof.
3. Complete the proof, making sure you have used both hypotheses.

Composition of injective functions (part 2)

Assume that all functions in this problem have domain $\mathbb{R}$.

Prove the following theorem:

Theorem

Let f and g be functions.

IF $f \circ g$ is injective, THEN g is injective.

Hint:

1. The theorem is of the form "$A \implies B$". In this case, it is easier to prove the equivalent theorem of the form "(not B) $\implies$ (not A)".
2. Having written this new statement, write down the definitions of the hypotheses and the conclusion.
3. Figure out the structure of the proof.
4. Complete the proof.