MAT137 - Week 9, Lecture 3

- **Reminder:** Next week is reading week! We did it!!

- **Reminder:** Problem Set 4 will be available on the course website shortly, and is due **Thursday 21 November, by 11:59pm**.

  Try to get a head start on it during reading week!

- Today's lecture will assume you have watched up to the end of playlist 4.

  For the first lecture after reading week, watch videos 5.1 through 5.4.
  Don't leave these ones until the last minute. Watch them twice, if you can.

Composition of injective functions (part 2)

Assume that all functions in this problem have domain $\mathbb{R}$.

Prove the following theorem:

Theorem

Let f and g be functions.

IF $f \circ g$ is injective, THEN g is injective.

Hint:

1. The theorem is of the form "$A \implies B$". In this case, it is easier to prove the equivalent theorem of the form "(not B) $\implies$ (not A)".
2. Having written this new statement, write down the definitions of the hypotheses and the conclusion.
3. Figure out the structure of the proof.
4. Complete the proof.

Composition of injective functions (part 2)

The following claim is false.

Claim

Let f and g be functions.

IF $f \circ g$ is injective, THEN f is injective.

Prove that this claim is false with a counterexample.

Our goal today

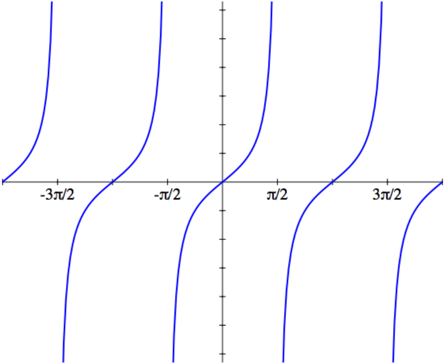
In videos 4.6 and 4.7 you saw a detailed treatment of the arcsin function. Specifically:

- How it is defined.
- What its domain and range are.
- What function it is the inverse of.
- How to compute $\sin(\arcsin(x))$ and $\arcsin(\sin(x))$ for different values of x.
- How to derive a formula for its derivative.

Today, we're going to do all of these things but for the arctan function.

The arctan function

Here is (part of) the graph of the tangent function.

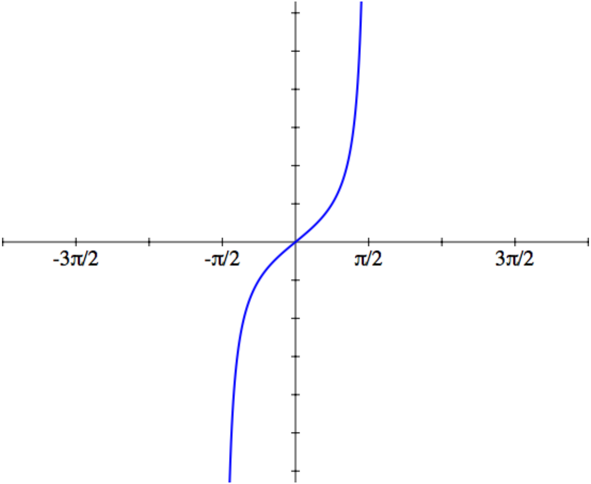


Question. Does this function have an inverse?

Problem. Find the largest interval containing 0 such that the restriction of tan to it is injective.

The arctan function (part 2)

We define arctan to be the inverse of the function with this graph:



The arctan function (part 3)

In symbols, that means we define the function arctan as the inverse of the function

$$g(x) = \tan x, \text{ restricted to the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

In other words, if $x, y \in \mathbb{R}$, then

$$\arctan(y) = x \iff \begin{cases} ??? \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ ??? \end{cases}$$

Problem 1. What should be where the question marks are?

Problem 2. What are the domain and range of arctan?

Problem 3. Sketch the graph of arctan.

The arctan function (part 4)

To remind you:

$$\arctan(y) = x \iff \begin{cases} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \tan x = y \end{cases}$$

Compute the following values:

1. $\arctan\left(\tan\left(1\right)\right)$
2. $\arctan\left(\tan\left(3\right)\right)$
3. $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
4. $\arctan\left(\tan\left(-6\right)\right)$
5. $\tan\left(\arctan\left(0\right)\right)$
6. $\tan\left(\arctan\left(10\right)\right)$

Derivative of arctan

We didn't get to this slide in class, but you should do this exercise when you have a chance.

Problem. Derive a formula for the derivative of arctan.

Hint: Start with the fact that

$$\text{for any } x \in \mathbb{R}, \; \tan\left(\arctan(x)\right) = x,$$

and differentate both sides with respect to x.

The process should be quite similar to the derivation of a formula for the derivative of arcsin that you saw in video 4.7.