MAT137 - Week 10, Lecture 2

- **Reminder:** Problem Set 4 is due **Thursday 21 November, by 11:59pm**.
- **Reminder:** Test 2 is scheduled for Friday 29 November.
- Today's lecture will assume you have watched up to and including video 5.6.

  For tomorrow's lecture, watch videos 5.7 through 5.9.

Common misconceptions

Suppose we know the following information about the function h:

- The domain of h is $(-4, 4)$.
- h is continuous at every point in its domain.
- h is differentiable on its entire domain, except at 0.
- $h'(x) = 0 \quad \iff \quad x = -1$ or 1.

Problem. What can you conclude about the maximum of h?

1. h has a maximum at $x = -1$, or 1.
2. h has a maximum at $x = -1, 0$, or 1.
3. h has a maximum at $x = -4, -1, 0, 1$, or 4.
4. None of the above.

What can you conclude?

Suppose we know the following information about the function f.

- f has domain $\mathbb{R}$.
- f is continuous
- $f(0) = 0$
- For every $x \in \mathbb{R}$, $f(x) \geq x$.

Problem. What can you conclude about $f'(0)$? Prove your answer.

Hint: Sketch a graph of what f might look like. Looking at your graph, make a conjecture.

To prove it, you can either imitate the proof of the Local EVT from Video 5.3, or use the Local EVT on a new function.

Note: The question does not say whether f is differentiable at 0, or anywhere else.

Zeroes of functions and their derivatives

For each part, if possible, construct a function f that is differentiable on $\mathbb{R}$ and such that:

1. f has exactly 2 zeroes and f' has exactly 1 zero.
2. f has exactly 2 zeroes and f' has exactly 2 zeroes.
3. f has exactly 3 zeroes and f' has exactly 1 zero.
4. f has exactly 1 zero and f' has infinitely many zeroes.

(A sketch of a graph is good enough for each part.)

Finding the number of roots of a function

Problem. Let f be the function defined by

$$f(x) = e^x - \sin x + x^2 + 10x.$$

How many zeroes does f have?