MAT137 - Week 10, Lecture 3

- **Reminder:** Problem Set 4 is due **Thursday 21 November, by 11:59pm**.
- **Reminder:** Test 2 is scheduled for Friday 29 November.
- Today's lecture will assume you have watched up to and including video 5.9.

  For next Tuesday's lecture, watch videos 5.10 through 5.12.

Finding the number of roots of a function

Problem. Let f be the function defined by

$$f(x) = e^x - \sin x + x^2 + 10x.$$

How many zeroes does f have?

Recall: During yesterday's class, we used Rolle's Theorem to prove that f has at most two roots.

A nice consequence of Rolle's Theorem

I skipped this slide in class, but it's a nice little exercise to practise.

The following theorem is missing some of its hypotheses. Fill in the missing hypotheses, then prove the theorem.

Theorem

Let $a < b$ be real numbers. Let f be a function defined on $[a, b]$.

IF

- *(Some conditions about continuity and differentiability.)*
- *f is **not** injective on $[a, b]$*

THEN $\exists c \in (a, b)$ such that $f'(c) = 0$.

1. Write the definition of "f is not injective on $[a, b]$". You will need it.
2. Recall the statement of Rolle's Theorem. You will need that too.
3. Do some rough work to understand why this is true.
4. Write the proof.

Using the MVT

Problem 1. Let f be a function that is continuous on $[0, 7]$ and differentiable on $(0, 7)$.

Suppose that $f(0) = -5$, and that $f'(x) \leq 10$ for all $x \in (0, 7)$.

What can you say about the value of $f(7)$?

Using the MVT (part 2)

Problem 2. Let $a, b \in \mathbb{R}$. Use the Mean Value Theorem to prove that

$$|\cos(a) - \cos(b)| \leq |a - b|.$$

Hint: You don't need to use any trig identities here. Just the MVT.

Proving difficult identities

Prove that there is some constant C such that for every $x \geq 0$,

$$\arcsin\left(\frac{1-x}{1+x}\right) + 2\arctan\left(\sqrt{x}\right) = C.$$

Hint: In other words, I'm asking you to prove that the function g defined by

$$g(x) = \arcsin\left(\frac{1-x}{1+x}\right) + 2\arctan\left(\sqrt{x}\right).$$

is constant on $[0, \infty)$.