MAT137 - Week 11, Lecture 2

- **Reminder:** Problem Set 4 is due **today, by 11:59pm**.
- **Reminder:** Test 2 is scheduled for next Friday 29 November. Please read the vocabulary list posted on the course website before the test.
- Problem Set B is on the website now. It contains material that is not covered by Problem Sets 1 through 4, but that is covered by Test 2. It is not to be submitted, but it is very good practise. Do these problems before Test 2.
- Today's lecture will assume you have watched up to and including video 6.2.

  For tomorrow's lecture, watch videos 6.3 through 6.7.

Use the MVT to prove this theorem

Theorem

Let $a < b$, and let f be a differentiable function defined on (a, b).

- *IF* $\forall x \in (a, b), f'(x) > 0$,
- *THEN f is increasing on (a, b).*

1. From the statement, write the structure of the proof.
2. If you use a theorem, did you verify the hypotheses?
3. Are there words in your proof, or just equations?

What is wrong with this "proof"?

Theorem

Let $a < b$. Let f be a differentiable function defined on (a, b).

- *IF $\forall x \in (a, b), f'(x) > 0$,*
- *THEN f is increasing on (a, b).*

"Proof".

- From the MVT, we know $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
- We know $b - a > 0$ and $f'(c) > 0$.
- Therefore $f(b) - f(a) > 0$, and so $f(b) > f(a)$.
- Therefore f is increasing.

This proof is:

- Total nonsense.
- The most common proof students write when we ask this on tests.

The classic optimization problem

(Every calculus student has to do this problem at least once.)

A farmer has 300m of fencing.

They want to create a rectangular enclosure, and then add extra fence to split the enclosure into two enclosures of equal area down the middle (to separate cows from sheep).

What is the area of the largest enclosure the farmer can build?

Another optimization problem

Find the (x, y)-coordinates of the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.