- **Reminder:** Test 2 is scheduled for next Friday 29 November. Please read the vocabulary list posted on the course website before the test.
- Problem Set B is on the website now. It contains material that is not covered by Problem Sets 1 through 4, but that is covered by Test 2. It is not to be submitted, but it is very good practise. Do these problems before Test 2.
- Today's lecture will assume you have watched up to and including video 6.7.

  For next Tuesday's lecture, watch videos 6.8 through 6.10.

Computations

Compute the following limits.

1. $\lim_{x\to 2} \frac{x^2+2x-6}{x^2+3x-10}$
2. $\lim_{x\to 0} \frac{e^{2x^2}-\cos x}{x \sin x}$
3. $\lim_{x\to\infty} \frac{x^2}{e^x}$
4. $\lim_{x\to\infty} (\sin x)\left(e^{1/x}-1\right)$
5. $\lim_{x\to\infty} x \sin \frac{2}{x}$
6. $\lim_{x\to\infty} x \cos \frac{2}{x}$
7. $\lim_{x\to 1} \left[(\ln x)\tan \frac{\pi x}{2}\right]$

Lessons from the previous slide

1. It's not indeterminate! Make sure to check before applying L'Hôpital.
2. Routine use of L'Hôpital's rule (twice), with lots of algebra.

   When you learn about Taylor series, you'll be able to compute this limit in a few seconds without writing anything down.
3. Routine use of L'Hôpital's rule (twice).

   On your own, check what happens with different limits of the form $\lim_{x \to \infty} \frac{x^a}{b^x}$, when $a > 0$ and $b > 1$.
4. It's not indeterminate!
5. Routine use of L'Hôpital's rule.

   Again, when you know about Taylor series, you we be able to write down this answer instantly.
6. It's not indeterminate!
7. Routine use of L'Hôpital's rule.

Careful with L'Hôpital's rule!

Problem 1. Evaluate this limit (no L'Hôpital's rule required).

$$\lim_{x\to\infty} \frac{x+\sin x}{x}$$

What is wrong with this argument

INCORRECT PROOF.

The top and bottom both $\to \infty$, so this is indeterminate of type $\frac{\infty}{\infty}$.
So:

$$\lim_{x\to\infty} \frac{x+\sin x}{x} \overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{1+\cos x}{1} = \lim_{x\to\infty} \left[1+\cos x\right].$$

The last limit doesn't exist, so $\lim_{x\to\infty} \frac{x+\sin x}{x}$ doesn't exist. □

Moral: Verify the hypotheses! L'Hôpital's rule can never tell you that a limit doesn't exist (except equalling $\pm\infty$).