- **Reminder:** Test 2 is scheduled for this Friday.
  Please read the vocabulary list posted on the course website before the test.

- Problem Set B is on the website now. It contains material that is not covered by Problem Sets 1 through 4, but that is covered by Test 2. It is not to be submitted, but it is very good practise. Do these problems before Test 2.

- Today's lecture will assume you have watched up to and including video 6.10.

  For Thursday's lecture, watch videos 6.11 and 6.12.

Careful with L'Hôpital's rule!

Last class you computed that $\lim_{x\to\infty} \frac{x+\sin x}{x} = 1.$

Then we saw this example of an incorrect use of L'Hôpital's rule. What was the flaw in this argument?

INCORRECT PROOF.

The top and bottom both $\to \infty$, so this is indeterminate of type $\frac{\infty}{\infty}$. So:

$$\lim_{x\to\infty} \frac{x+\sin x}{x} \overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{1+\cos x}{1} = \lim_{x\to\infty} \left[1+\cos x\right].$$

The last limit doesn't exist, so $\lim_{x\to\infty} \frac{x+\sin x}{x}$ doesn't exist. □

Careful with L'Hôpital's rule! (part 2)

Problem 2. Try to use L'Hôpital's rule to compute this limit.

$$\lim_{x\to\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

What happens?

Now, what does the limit actually equal?

Moral: Sometime's L'Hôpital's rule can apply to a limit, but not tell you anything helpful.

It's just another tool you can use, not a magic solution to all indeterminate limits.

Some computations, as a warm up.

Compute the following limits:

1. $\lim_{x \to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x} \right]$

2. $\lim_{x \to \infty} \left[\ln(x+2) - \ln(3x+4) \right]$

3. $\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$

4. $\lim_{x \to 0} \left[1 + 2\sin(3x) \right]^{4\cot(5x)}$

Proving something is an indeterminate form

We didn't see this in this lecture and probably won't see it in class, but it's a nice exercise to convince yourself about why these are *indeterminate* forms.

In video 6.9 you saw a proof that 1^∞ is an indeterminate form.

Problem 1. Prove that 0^0 and ∞^0 are indeterminate forms.

Problem 2. Is either of 0^∞ or ∞^∞ an indeterminate form?

If yes, prove it. If not, what is the value of such a limit?