MAT137 - Week 12, Lecture 2

- **Reminder:** Test 2 is tomorrow.
  Please read the vocabulary list posted on the course website before the test.

- Problem Set B is on the website now. It contains material that is not covered by Problem Sets 1 through 4, but that is covered by Test 2. It is not to be submitted, but it is very good practise. Do these problems before Test 2.

- Today's lecture will assume you have watched up to and including video 6.12.

  For tomorrow's lecture, watch videos 6.13 through 6.16.

A preview of Taylor series

Let f be a function with domain $\mathbb{R}$. Assume f is differentiable at 0 as many times as you need.

1. Find $a, b \in \mathbb{R}$ such that

$$\lim_{x \to 0} \frac{f(x) - [a + bx]}{x} = 0$$

2. Find $a, b, c \in \mathbb{R}$ such that

$$\lim_{x \to 0} \frac{f(x) - \left[a + bx + cx^2\right]}{x^2} = 0$$

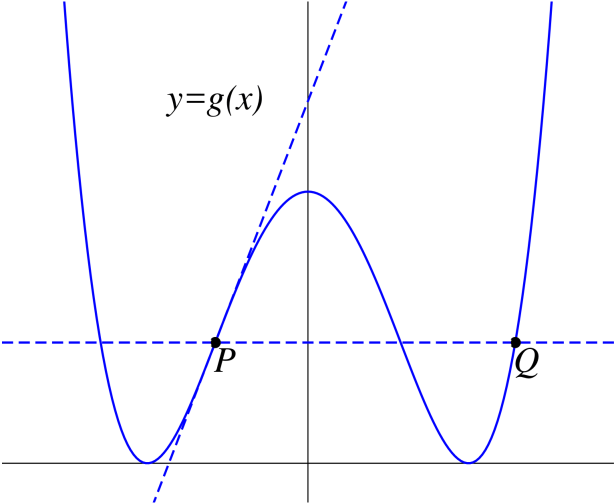
3. Let $N \in \mathbb{N}$. Find a polynomial P_N such that

$$\lim_{x \to 0} \frac{f(x) - P_N(x)}{x^N} = 0$$

Find the coordinates of P and Q

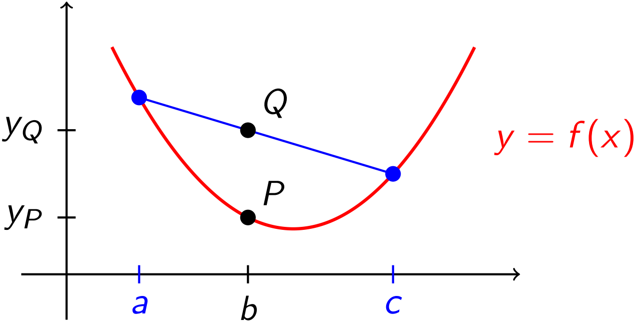
A nice practise problem that I skipped over in class.

$$g(x) = x^4 - 6x^2 + 9$$



Alternate definition of concave up

Let f be a function defined on an interval I.
In Video 6.11 you learned that an alternative way to define "f is concave up on I" is to say that "the secant segments stay above the graph".



Rewrite this as a precise mathematical statement of the form

"$\forall a, b, c \in I, \quad a < b < c \implies$ an inequality involving f, a, b, c"

Monotonicity and concavity

Let $f(x) = xe^{-x^2/2}$.

To save you time, its first two derivatives are:

$$f'(x) = -e^{-x^2/2}\,(x^2 - 1) \qquad\qquad f''(x) = e^{-x^2/2}\,x\,(x^2 - 3).$$

1. Find the intervals where f is increasing or decreasing, and its local extrema.
2. Find the intervals where f is concave up or concave down, and its inflection points.
3. Calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
4. Using this information (and only this information), try to sketch the graph of f.