MAT137 - Week 13, Lecture 1

- **Reminder:** This is our last lecture for the fall term!

- Problem Set 5 will be on the website shortly. It's due on the first Thursday of the winter term.

  You have the whole holiday break to work on it, but don't leave it until the last minute!

- Please fill out our mid-year course feedback form.

  It shouldn't take you much more than 10 minutes, and your feedback really is valuable to me, the other instructors, and the TAs.

- Today's lecture will assume you have watched all the videos on playlist 6.

Unexpected asymptotes

The function

$$f(x) = x + \sqrt{x^2 + x}$$

has two asymptotes.

Problem. Find them.

Lots of asymptotes

Consider the function $f(x) = \dfrac{x-1}{\sqrt{4x^2-1}}$.

Here are its first two derivatives, fully factored:

$$f'(x) = \frac{4x-1}{(4x^2-1)^{3/2}} \qquad f''(x) = -\frac{4(8x^2-3x+1)}{(4x^2-1)^{5/2}}.$$

1. Determine the domain of f.
2. This function has *four* asymptotes. Find them!
3. Use f' to study its monotonocity.
4. Use f'' to study its concavity.
5. Sketch the graph of f.

We didn't see anything after this slide in class on Tuesday.

However, since I think these sorts of problems are great for practising and testing your understanding, I'm posting a few more for you to play with during the break. Enjoy!

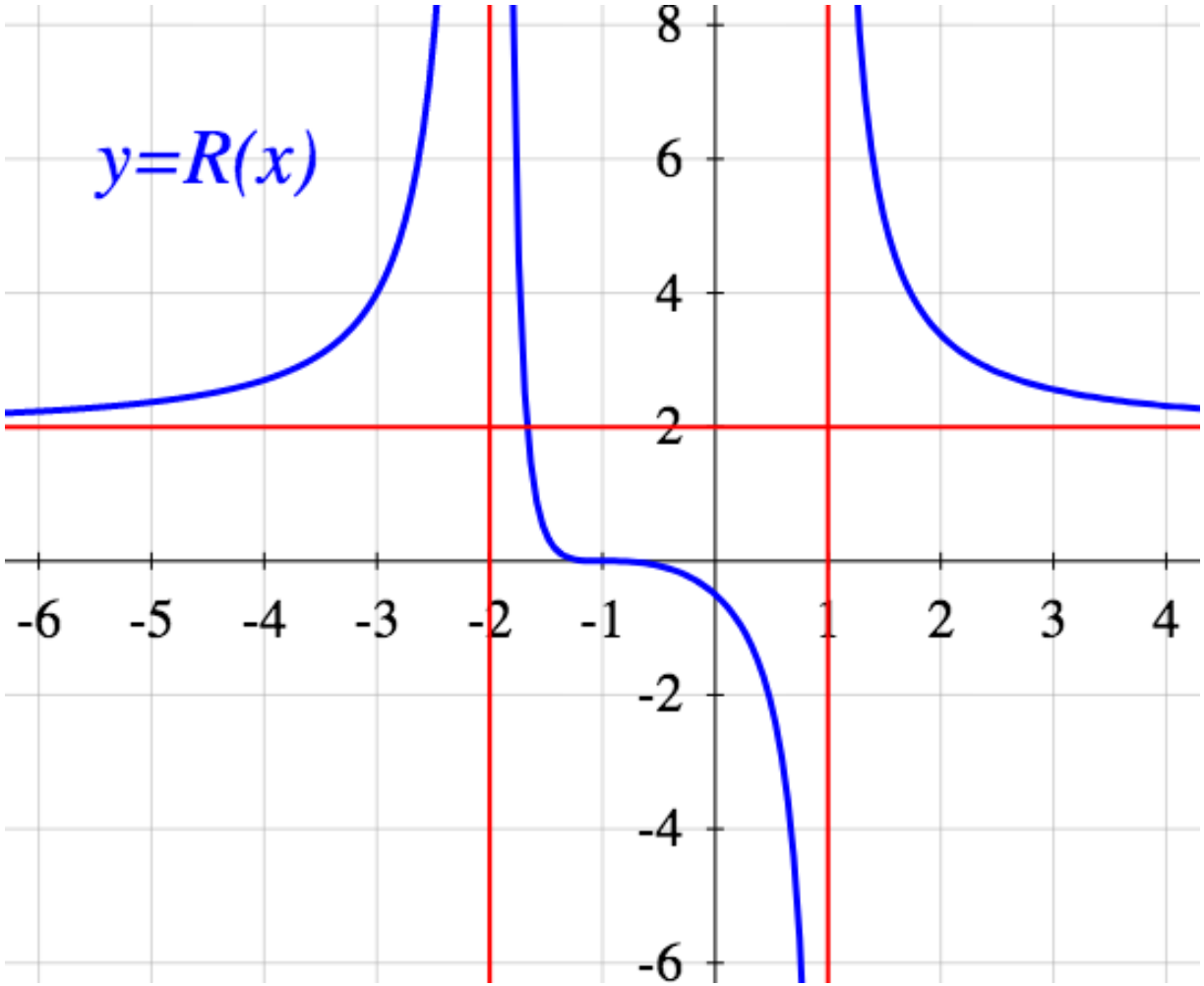
An equation from the asymptotes

Construct a function f that satisfies all the following conditions at the same time.

- f is a rational function (this means it is a quotient of polynomials).
- The line $y = 1$ is an asymptote of the graph of f.
- The line $x = -1$ is an asymptote of the graph of f.

Backwards graphing

This is the graph of $y = R(x)$. R is a rational function (a quotient of polynomials). From this graph alone, find the equation of R.



A tricky function to graph

The function $f(x) = xe^{1/x}$ is weird. To save you time, here are its derivatives:

$$f'(x) = \frac{x-1}{x}e^{1/x} \qquad f''(x) = \frac{e^{1/x}}{x^3}$$

1. Examine the behaviour f as $x \to \pm\infty$.
   There is an asymptote, but it's tricky to see.
2. Carefully examine the behaviour of f as $x \to 0^+$ and $x \to 0^-$.
   They are very different.
3. Use f' to study its monotonocity.
4. Use f'' to study its concavity.
5. Sketch the graph of f.