MAT137 - Term 2, week 1, lecture 1

- **Reminder:** Problem Set 5 is due this Thursday at 11:59pm.
- Problem set 6 is due a week and a half later, on **Monday**, 20 January.
- Today's lecture will assume you have watched up to and including video 7.2.

  For Thursday's lecture, watch videos 7.3 and 7.4.

  For Friday and next Tuesday you're going to need to watch up to video 7.9. This content might be a bit tricky so the earlier (and the more times) you watch it, the better.

Sigma notation is just a way of making it easier to express long summations in a more compact form.

It's just notation. No new concepts here. Just notation to make it simpler to write some things.

$\sum$ is the Greek letter *sigma*, which is Greek version of "S". ("S" for "sum".)

You should spend a few minutes at some point practising how to write sigmas. Seriously.

Sigma notation

If you've ever done any programming, sigma notation can be thought of like a very simple *for* loop.

For example, the expression

$$\sum_{i=1}^{7} a_i$$

essentially executes the following pseudocode:

```
sum = 0
FOR i = 1 to 7
    sum = sum + a_i
    i = i + 1
RETURN sum
```

It equals $a_1 + a_2 + a_3 + \cdots + a_7$.

Sigma notation exercise

Consider the following sum written in sigma notation:

$$\sum_{j=0}^{N} \frac{x^j}{2j+1}.$$

Does the value of this expression depend on...

1. ...x only?
2. ...N only?
3. ...j only?
4. ...x and j?
5. ...j and N?
6. ...x and N?

Sigma notation exercise

Write the following quantities as a single sum in sigma notation. There may be many ways to do each of them.

1. $2^7 + 3^7 + 4^7 + 5^7 + 6^7 + 7^7$
2. Same as the previous one, but start your sum from $i = 107$
3. $3 + 5 + 7 + 9 + \cdots 75 + 77$
4. $\displaystyle\sum_{i=1}^{100} a_i - \sum_{i=1}^{77} a_i$
5. $\cos(0) - \cos(2) + \cos(4) - \cos(6) + \cos(8) - \cdots \pm \cos(2N)$
6. $2 + \dfrac{5}{2} + \dfrac{10}{3} + \dfrac{17}{4} + \dfrac{26}{5} + \dfrac{37}{6} + \dfrac{50}{7}$
7. $-\dfrac{2x^4}{3!} + \dfrac{3x^5}{4!} - \dfrac{4x^6}{5!} + \cdots - \dfrac{98x^{100}}{99!}$

Sigma notation exercise

Didn't see this one in class, but it's a nice exercise.

Consider the following sum:

$$3 + 9 + 15 + 21 + 27 + 33 + \cdots + 297 + 303$$

Which of the following expressions represents the value of this sum (there may be more than one)?

1. $\displaystyle\sum_{n=1}^{51} 3(2n+1)$
2. $\displaystyle\sum_{n=1}^{51} 3(2n-1)$
3. $\displaystyle\sum_{i=0}^{50} 3(2i+1)$
4. $\displaystyle\sum_{i=0}^{50} 3(2n+1)$
5. $\displaystyle 3\sum_{n=0}^{50} (2n+1)$
6. $\displaystyle 3\sum_{n=7}^{57} (2n-13)$

Telescoping sums

Compute the precise value of the following expression:

$$\sum_{n=1}^{2020} \left[\frac{1}{n} - \frac{1}{n+1} \right]$$

Hint: If you're stuck, try writing out the first few terms of the summation.