MAT137 - Term 2, week 1, lecture 2

- **Reminder:** Problem Set 5 is due today at 11:59pm.
- Problem set 6 is due on **Monday**, 20 January.
- Today's lecture will assume you have watched up to and including video 7.4.

  For Friday and next Tuesday watch videos 7.5 through 7.9.

  This content might be a bit tricky so the earlier (and the more times) you watch it, the better.

Infima and suprema exercise

For each of the following sets of real numbers

- find its supremum or convince yourself it does not exist;
- do the same for the infimum;
- find the maximum and minimum, if they exist.

1. $A_1 = (0, 7)$
2. $A_2 = (0, 7]$
3. $A_3 = \{7, 8, 9\}$
4. $A_4 = \{ x \in \mathbb{R} : x < 0 \text{ or } x \geq 7 \}$
5. $A_5 = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\} = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$
6. $A_6 = \{\dots, \frac{1}{343}, \frac{1}{49}, \frac{1}{7}, 1, 7, 49, 343, \dots\} = \{ 7^n : n \in \mathbb{Z} \}.$

Empty set questions

1. Does $\emptyset$ have an upper bound ?
2. Does $\emptyset$ have a supremum?
3. Does $\emptyset$ have a maximum?

Recall:

Let $A \subseteq \mathbb{R}$. Let $a \in \mathbb{R}$.

- a is an **upper bound** of A means: $\forall x \in A, x \leq a$.
- a is the **least upper bound** (lub) or **supremum** (sup) of A means
  - a is an upper bound of A, and
  - there are no smaller upper bounds.

Question. Does any of this contradict the Least Upper Bound Principle?

Infima and suprema exercise

Recall again that M is the supremum of a set A if...

1. ...M is an upper bound of A;
2. ...and there are no smaller upper bounds of A.

With this in mind, **assume M is an upper bound for a set A.** Which of the following is equivalent to "M is the supremum of A"?

1. If L is an upper bound of A, then $L \geq M$.
2. $\forall L \geq M$, $\quad L$ is an upper bound of A.
3. $\forall L \leq M$, $\exists x \in A$ such that $L < x$.
4. $\forall L < M$, $\exists x \in A$ such that $L < x$.
5. $\forall L < M$, $\exists x \in A$ such that $L < x \leq M$.
6. $\forall \varepsilon > 0$, $\exists x \in A$ such that $M - \varepsilon < x \leq M$.