MAT137 - Term 2, week 1, lecture 3

- **Reminder:** Problem set 6 is due on **Monday**, 20 January.
- Today's lecture will assume you have watched up to and including video 7.9.

  For next Tuesday, watch videos 7.5 through 7.9 again!

  This content might be a bit tricky so the earlier (and the more times) you watch it, the better.

Infima and suprema exercise

Recall again that M is the supremum of a set A if...

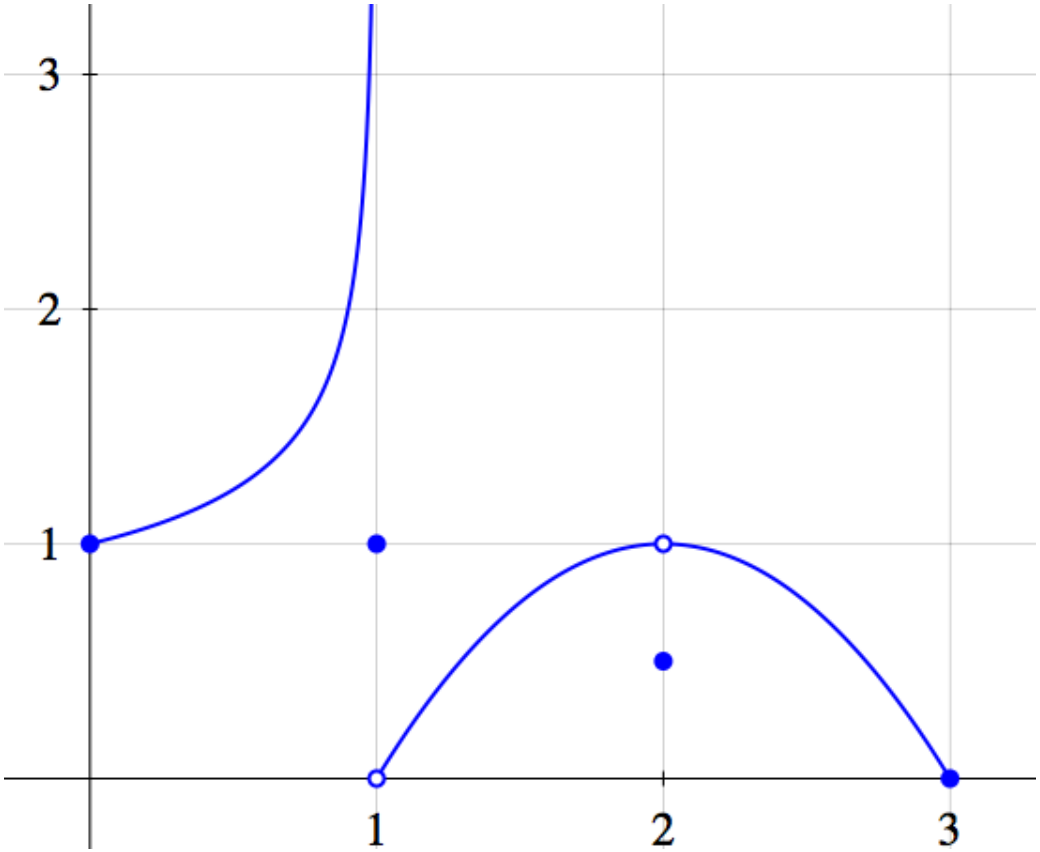
1. ...M is an upper bound of A;
2. ...and there are no smaller upper bounds of A.

With this in mind, **assume M is an upper bound for a set A.** Which of the following is equivalent to "M is the supremum of A"?

1. If L is an upper bound of A, then $L \geq M$.
2. $\forall L \geq M$, $\quad L$ is an upper bound of A.
3. $\forall L \leq M$, $\exists x \in A$ such that $L < x$.
4. $\forall L < M$, $\exists x \in A$ such that $L < x$.
5. $\forall L < M$, $\exists x \in A$ such that $L < x \leq M$.
6. $\forall \varepsilon > 0$, $\exists x \in A$ such that $M - \varepsilon < x \leq M$.

Infima and suprema of functions exercise

Consider the following function f.



1. $[0, 3]$
2. $[0, 1)$
3. $[1, 2]$
4. $(1, 2)$
5. $(1, 2]$
6. $[2, 3]$

On each of the given domains, determine whether f is bounded, its infimum and supremum (if they exist), and its maximum and minimum (if they exist).

Warm up: partitions

Which of the following are partitions of $[0, 2]$?

1. $[0, 2]$
2. $(0, 2)$
3. $\{0, 2\}$
4. $\{1, 2\}$
5. $\{0, 1.5, 1.6, 1.7, 1.8, 1.9, 2\}$

Warm up: lower and upper sums

Let $f(x) = \cos x$.

Consider the partition $P = \{0, 1, 2, 4\}$ of the interval $[0, 4]$.

Problem. Compute $L_P(f)$ and $U_P(f)$.

Joining partitions

Let f be a bounded function defined on some interval $[a, b]$. Let P and Q be partitions of $[a, b]$.

Assume

$$L_P(f) = 2, \quad U_P(f) = 6,$$
$$L_Q(f) = 3, \quad U_Q(f) = 8.$$

Problem.

1. Is $P \subseteq Q$?
2. Is $Q \subseteq P$?
3. What can you say about $L_{P \cup Q}(f)$ and $U_{P \cup Q}(f)$?